THE INTERNAL AND EXTERNAL LOGICAL STRUCTURE OF THE LEGAL SENTENCES

Hajime Yoshino

Professor of Law Emeritus, Meiji Gakuin University
1-2-37 Shirokanedai, Minato-ku, Tokyo, JP
hyoshino@ls.meijigakuin.ac.jp; http://www.meijigakuin.ac.jp/ yoshino/

Abstract: *To build useful legal reasoning systems for the future society and to also approach the genuine science of laws, the logical structure of the laws should be precisely clarified so that the relevant legal knowledge bases (LKB) and exact theories of laws can be constructed. This paper clarifies the internal and the external logical structure of legal sentences. The internal structure is what kinds of internal structures the minimal units of legal sentences themselves have. The external structure of legal sentences is how those legal sentences are united with each other to constitute a legal system.*

1. Introduction

In the well-developed future society, Artificial Intelligence systems (AI) for almost all laws should be provided by governments and/or the private sector so that people (lawyers as well as citizens) are able to use them to simulate what should be the result of the application of the relevant laws to the problems they are confronted with. To achieve such legal reasoning systems, and to also approach the genuine science of laws, the logical structure of the laws should be precisely clarified, so that the relevant legal knowledge bases (LKB) as well as an exact theory of law can be constructed. To construct such legal knowledge bases and theories of law, the laws should be analyzed into simple minimal units and reconstructed as the logical connections of such minimal units.

This paper tries to clarify and make precise the logical structure of laws in terms of the internal and external structure of legal sentences. The internal structure of legal sentences is what kinds of internal structures the minimal units of legal sentences have. The external structure of legal sentences is how those legal sentences are united with each other to constitute a legal system. Through these clarifications by Logical Jurisprudence (LJ), the basis of logical formalization of laws which enables us to develop LKB's of a total system of legal reasoning as well as the science of law for future societies will be provided.

2. The Traditional Legal Theories on the Structure of Laws

The law has traditionally been developing together with legal theories. People's understanding of laws is more or less based on those theories. Therefore, it would be wrong for us to ignore them when building LKB's, but we should examine if their approaches are useful and in what points they are limited for LKB's. We should find ways to overcome their limitations. Below, we will consider HANS KELSEN's and H.L.A. HART's legal theories on the structure of laws.

HANS KELSEN regards legal norms, which is to be conceived as meaning of legal sentences, as an object of scientific cognition of law and tries to develop a theory to clarify the system of legal norms.

Kelsen understands the legal norm as «a hypothetical judgment that expresses the specific linking of a conditioning material fact with a conditioned consequence». Thus, the legal norm becomes the law-sentence (Rechtssatz), which shows the basic form of statutes.[1]

According to Kelsen, law consists of primary norms and secondary norms. Kelsen defines these as follows: primary norms are norms which describe: «under the condition of the opposite behavior, a coercive act should be occurred as consequence». The norm establishing a sanction-avoiding behavior would be a secondary legal norm.[2] Kelsen considers the sanction as an essential element of law. Therefore, he regards legal norms which regulate people's behaviors as secondary norms and legal norms which regulate sanctions given to the people who breach the relevant secondary norms as primary norms.

H.L.A. Hart tries to clarify the concept and structure of law in terms of internal and external aspects on the one hand, and primary and secondary rules on the other hand. The observer's view of a set of rules is based on an external aspect and the view of members who accept and use those rules belongs to the internal aspect. Through his introduction of the internal aspect, Hart enables us to explain law not simply to be sanction-threatening but rather obligation-imposing.[3] According to Hart, «while primary rules are concerned with the actions that individuals must or must not do, these secondary rules are all concerned with the primary rules themselves.[4] They specify the ways in which the primary rules may be conclusively ascertained, introduced, eliminated and varied and the fact of their violation conclusively determined.»[5] He explains law as the union of primary and secondary rules.[6] Thus, Hart has made great contributions to the clarification of the structure of law. However, his analysis remains a rough descriptive-sociological analysis of people's behaviors related to legal rules. It lacks a precise linguistic or logical analysis of legal rule sentences as linguistic objects and their mutual relations. Thus, a precise linguistic and logical analysis of the structure of law, that Hart's theory would suggest with his conception of primary and secondary rules and their union, is needed to enable us to develop a total system of legal reasoning in the future society. This will be addressed in this paper in terms of LJ.

3. The Internal Structure of Legal Sentences – Three Fundamental Alternative Sorts of Legal Sentences

From the internal point of views of legal sentences, the author will clarify the internal logical structure of law in terms of three fundamental alternative types of legal sentences: (1) legal rule sentences and fact sentences, (2) legal element sentences and complex sentences, and (3) legal object sentences and meta-sentences.

3.1. Legal Rule and Fact Sentences

(1a) **legal rule sentences** have the following internal syntactic structure:

$\forall X(a (X) \leftarrow b (X)).$[7]

The logical structure of legal rule sentences that have the legal requirement and the legal consequence corresponds to this:

[1]   Kelsen (note 1).
[2]   Kelsen (note 1), p. 30. English translation (not 1), p. 30.
[3]   Shapiro, Scott J., What is the Internal Point of View?, Faculty Scholarship Series, 200,. Paper 1336, p. 1157. It is good that Shapiro highly evaluates Hart's internal aspect because it enables the character of legal rules (not merely as sanction-affording but) obligation-imposing. However, we notice that the sanction-affording aspect is essential for laws in comparison with morals on the one hand and that the sanction-affording rules can and should be reduced to the obligation-imposing on the other hand.
[4]   That secondary rules are concerned with primary rules themselves means that the former are meta-rules of the latter rules. However, it is a pity that H.L.A. Hart does not refer to the nature of secondary rules as meta-rules.
[5]   Hart, H.L.A., The concept of law, 2nd edition, Oxford University Press 1994, p. 80.
[6]   Hart (note 6), p. 82–92.
[7]   This is a predicate logical formula in which a variable is to be represented in a capital letter.

$\forall X(legal\_consequence(X) \leftarrow legal\_requirement(X))$

(1b) **Legal fact sentences** have the following syntactic structure:

b(x1).

legal_requirement(x1)).

3.2. Legal Element and Complex Sentences

Legal element sentences mark the minimal units of legal sentences, whereas complex legal sentences are the combination of legal sentences, typically with a unique name for the whole. For example, «A contract is concluded at the moment when an acceptance of an offer becomes effective» (Article 23 of the United Nations Convention on Contracts for the International Sale of Goods [CISG]) is a legal element sentence. The CISG itself, its parts, its chapters, most of its articles are legal complex sentences. The internal logical structure of legal element sentences and the complex legal sentence in the above example can be represented as follows:[8]

cisg_article(23): is_concluded(_,[contract(C,[A,B,M,S]),T]) ←
become_effective(_,[acceptance(Ac,[A,B,Of,S2]),T]) &
is_effective(_,[offer(Of,[A,B,S1]),T1]).[9]

cisg: complex_sentence(cisg,[
part(part1,[...]),
part(part2,[article(14,[(1),(2)]),..., article(23)],..., article(29,[(1),(2)])]),
part(part3,[...]),
part(part4,[...])])

3.3. Legal Object and Meta Sentences

(a) Legal object sentences regulate the obligation of people's behavior so that it guides people's thought to do their actions in the way prescribed by the legal sentence when they read or hear it. Therefore, the logical formulae of legal object sentences should have the predicate representing «obligation,»[10] and the terms for a person as the norm-subject (assigning who is obligated) and for an action as the norm-object (assigning what is obligated). The legal object sentence has two more sorts according to the difference between rule and fact sentences: legal object fact sentences and legal object rule sentences.

Legal object fact sentences have, for example, the following syntactic structure:

is_obligatory(Person,Action).[11]

is_obligatory('Bernard',pay('Bernard', 'Anzai', price($50,000,goods), t22_02_2018).[12]

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[8] This formula is a CPF. CPF is the abbreviation of Compound Predicate Formula, which is a logical representation developed by the author for his legal reasoning systems. As regards the foundation of CPF, see: Yoshino, Hajime, On the Logical Foundation of Compound Predicate Formulae for Legal Knowledge Representation, in: Artificial Intelligence and Law, Vol. 5, No. 1–2 1997, pp. 77–96. For a simple explanation of CPF: see: Sakurai, Seichiro/Yoshino, Hajime, Identification of Implicit Legal Requirements with Legal Abstract Knowledge, in: Proceedings of the Fourth International Conference on Artificial Intelligence and Law, ACM, 1993, pp. 298–305. All variables used in this formula are bound by universal quantifiers which are eliminated.

[9] In this rule, «the offer is effective» is set as its second requirement. The CISG article 23 itself does not directly express that «the offer is effective» is a requirement for a contract to be concluded. Through a creative systematic reasoning from the related articles, this requirement is added in this representation of the rule. Cf.: Sakurai/Yoshino (note 9).

[10] The predicate is not restricted to the noun «obligation.» Other predicates which represent the conception of the obligation are available, e.g., «is obligatory,» «must,» «has to,» and so on.

[11] «*Person*» is used for a variable assigned to an item of person and «*Action*» to an item of action.

[12] This formula to be read: «It is obligatory for Bernard to pay Anzai the price of goods $50,000 on February 22nd 2018.»

Legal object rule sentences should have an object fact sentence in the part of legal consequence of the relevant legal rule sentence. For example, they have the following syntactic structure:[13]

$\forall X,Z(is\_obligatory(X,Z) \leftarrow$

    person(X) & action(Z)).

$\forall X,(is\_obligatory(X,Z) \leftarrow$

    buyer(X, sale(S, X,Y,G)) & pay(Z,X,seller(Y,S),price(P,G)))).[14]

(b) Legal meta-sentences regulate the validity of legal sentences. Therefore, their logical formulae should have the predicate representing «validity,»[15] and the terms for legal sentences, whose validity is in question, and the terms for the scope of validity (in which scope of time, places, people and matters the sentence is valid). The legal object sentence has two more sorts according to the difference between rule and fact sentences: legal meta-fact sentences and meta-rule sentences.

Legal meta-fact sentences have, for example, the logical structure of the following fact sentence in which a legal sentence identifier (*s1*) constitutes a term of predicate formulae. (In these examples, the scope of the validity is restricted only to time).

is_valid(s1,23_02_2017).

become_valid((s1,23_02_2016).

become_null((s1,23_02_2018).

Legal meta-rule sentences have, for example, the logical structure of the following rule sentence in which a legal sentence variable (*S*) constitutes a term of predicate formulae:

[r0]:$\forall S \forall T(is\_valid(S,T) \leftarrow \rightarrow$

    become_valid(S,T1) & before_or_same_time(T1,T) &

    not((become_null(S,T2]) & before_or_same_time(T2,T)))).

In positive laws, we can find many legal rule sentences which determine if sentences become valid or become null (or is terminated). However, we cannot find any legal rule sentence which directly determines if a sentence is valid or not. Through the analysis of positive laws and the construction of legal reasoning systems, the author has found the legal meta-rule sentence *[r0]* above. Whenever the validity of a legal sentence is questioned, this rule sentence is implicitly applied to determine whether the legal sentence is valid[16]. In this sense, this meta-rule sentence is to be called as the Most Fundamental Legal Meta-Rule Sentence (MFLMRS). To determine whether the first requirement of this rule sentence «*become_valid(S,T1)*» and the second requirement «*become_null(S,T2)*» are true, there are several implicit fundamental legal meta-rule sentences and a great number of positive legal meta-rule sentences. The further examples will be presented in the next chapter.

4. The External Structure of Legal Sentences – the Union of Legal Sentences

The author will clarify the external structure of legal sentences in terms of the logical union of such legal sentences.

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[13]   This is the approach in which the deontic term «obligatory» is represented with a predicate. We think that «Deontic Logic» is not necessary to logically formalize legal rule sentences.

[14]   This formula is to be read as follows: «If X is a buyer of the sale S of goods G between X and Y and Z is to pay the seller Y the price P, then it is obligatory for X to do Z.»

[15]   The predicate is not restricted to the noun «validity.» Other predicates which represent the conception of the validity are available, e.g., «is valid,» «is become valid,» «become null,» «is terminated,» and so on

[16]   People apply rules to a problem usually without checking the validity of the rules because they believe it or presuppose that the relevant rules are valid. However, to put it precisely, one should always check whether the rule is valid whenever one applies a rule. This will be necessary especially for a computer based legal reasoning.

As a preparatory exercise, the author would like to discuss the way to logically represent that two legal sentences exist in the legal world of discourse. There are two ways:

(a) to write two sentences separately into two logical formulae like this:

∀X(a(X)←b(X)).

∀X(a(X)□c(X))., or

(b) to write two sentences connected by the conjunction of the relevant legal sentences into one logical formula like this:

∀X(a(X)←b(X)) &∀X(a(X)←c(X)).

If we presuppose the world of inference, i.e., of the inference by practitioners and interpreters of law, or of the inference in legal reasoning systems, then we should represent two legal rules separately as (a), without using any conjunction operator as (b). In the following, the author will apply the way of representation (a).

Now let us discuss the logical structure of the union of legal sentences. According to three alternative structure-types of legal sentences explained above as their internal structures, the author clarifies briefly the union of legal sentences in three ways.

4.1. The Union of Legal Rule and Fact Sentences

The logical union of a certain type of legal rule and fact sentences realizes a logical inference automatically when they are located in the practitioner's memory or in the computer software memory like Prolog in legal reasoning systems:

(1) ∀X{a(X)←b(X)}

(2) b(x1)

    ―――――――――

(3) a(x1)

From a legal rule sentence *(1)* and a legal fact sentence (2), a legal fact sentence *(3)* is logically deduced from the logical inference rule Modus Ponens.

4.2. The Union of Legal Element and Complex Sentences

Legal element sentences are united as a group in a legal complex sentence and legal element sentences in a legal complex sentence can be treated at once in the application of the relevant legal meta-rule sentences. The examples of such legal meta-rule sentences will be presented later as the *r01* and *r02*.

4.3. The Union of Legal Object and Meta Sentences

As described above, legal object sentences regulate the obligations of people's actions and a legal meta-sentence regulates the validity of legal sentences. According to LJ, Hart's «primary rules» are to be considered as legal object rule sentences and his «secondary rules» as legal meta-rule sentences. Hart explains law as the union of primary and secondary rules as seen above. How are they united to constitute the legal system as a whole? He does not logically analyze the union of primary and secondary rules. The author will logically clarify the structure of the union of legal object and meta-sentences, which are similar to Hart's primary and secondary rules.

Suppose that there are two legal sentences, where *s1* is a legal object sentence and *s2* is a legal meta-sentence:

[s1]: A must pay $55,000 to B by February 23, 2018.

[s2]: 'A must pay $55,000 to B by February 23, 2018' is valid on January 20, 2018.

The logical representation of the sentences above and their union can be represented as follows:

[s1]: must('A',pay('A', $55,000,B,time(T,by(23_02_2018))).

[s2]: is_valid(must('A',pay('A', $55,000,B,time_by(T,23_02_2018)),time(20_01_2018)).

If the sentence identifier is used instead of the sentence content itself, *s2* can be represented more simply as:

[s2]: is_valid(s1,time(20_01_2018)).

Here, the legal object sentence *s1* is treated as a term in the sentence *s2* whose predicate is «*is_valid*». Therefore, the sentence *s2* is a meta-sentence to the sentence *s1*. A legal object sentence and a meta-sentence are logically united in this way. This is the logical structure of the union of legal object sentences and meta-sentences.

To prove that *s2* is true, the *MFMRS (r0)* above is to be applied. To determine whether its first requirement and second requirement is fulfilled, there are further fundamental legal meta-rule sentences as well as positive legal rule sentences, through whose application it is determined whether *s1* is true. This inference is called «legal meta-inference» by LJ. What kind of legal meta-rule sentences are to be applied in the legal meta-inference to determine the first requirement of *MFMRS (r0)*?

The author has found that the following fundamental legal meta-rule sentences are implicitly valid to determine the first requirement of MFLMR *(r0)*, i.e., *become_valid(S,T1)*:[17]

[r01] A legal element sentence S becomes valid at time T, if

> *S is an element sentence of legal complex sentence CS, and*

> *legal complex sentence CS becomes valid at time T.*

[3AA1]: A legal complex sentence becomes valid at time T, if

> *the legal complex sentence is formed at time T1 and T1 is before or at the same time as T, and*

> *it is not the case that the complex sentence is invalid, and*

> *((the complex sentence entails the beginning time of effectiveness and*

>> *the beginning time has come at T) or*

> *(the complex sentence entails a condition of the effectiveness and*

>> *the condition is fulfilled at time T)) or*

> *T is T1).*

The first requirement of the rule *3AA1* above shows that the **formation of a legal complex sentence** is to be differentiated from **the validity of the legal complex sentence**. The former constitutes a condition of the latter. However, it is not a sufficient condition but a necessary condition for the validity of a legal complex sentence.[18] As each contract, degree, statute, constitution and convention is a legal complex sentence, legal meta-rule sentences which regulate the formation of a contract, a degree, a statute, a constitution, and a convention can be applied to determine whether the first requirement of the rule sentence *3AA1* above is fulfilled or not. For example, the following legal meta-rule sentences are applicable:

[CISG Article 23]: «A contract is concluded at the moment when an acceptance of an offer becomes effective in accordance with the provisions of this Convention.»

[The Constitution of Japan Article 59] «A bill becomes a law on passage by both Houses, except as otherwise provided by the Constitution.»

The second requirement of the rule *3AA1* is that the legal complex sentence is not invalid. If the relevant legal complex sentence is invalid, even if the sentence is formed as a legal complex sentence, it cannot become valid. The following are the examples of positive legal meta-rule sentences which regulate the invalidity of legal sentences:

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[17] To save space in the paper, in the following, not predicate logical formulae but their natural language representations will be given to express the relevant rule sentences.

[18] It is important to differentiate the concept of formation of law from the validity of law.

[The Constitution of Japan Article 98] This Constitution shall be the supreme law of the nation and no law, ordinance, imperial rescript or other act of government, or part thereof, contrary to the provisions hereof, shall have legal force or validity.

[Civil Code Article 90] A juristic act with any purpose which is against public policy is void.

In some cases, a legal element sentence alone becomes valid independently from a complex sentence. One of them is the fundamental legal meta-rule sentence regulating the relation between rights and duties. The following meta-rule sentences must be implicitly valid as a fundamental legal meta-rule sentence in the legal world. Positive laws have been issued on the assumption that this rule sentence is valid:

[3aa2]: «B must do Z» becomes valid at time T, if

   «A may require B to do Z» is valid at T &

   A requires B to do Z at T.[19]

The author analyses and formalizes legal rules which confer powers to create obligations in the sense of H.L.A. HART[20] as legal meta-rule sentences. «To create an obligation» means «to make a legal object sentence describing an obligation become valid».

A legal sentence illustrating that A has a right to require B to do Z is a legal meta-sentence which represents the state of affairs that one has a power to make the sentence «B must do Z» become legally valid by executing the right.[21]

What kind of legal meta-rule sentences are to be applied in the legal meta-inference to determine the second requirement of *MFMRS (r0)*?

The second requirement or *r0* is that it is not the case that the relevant legal sentence «becomes null» before or at the same time when the validity of the relevant legal sentence is questioned. The author has found several fundamental legal meta-rule sentences regulating how legal sentences become null. The following is an example.

[r02]: A legal sentence becomes null if

   it is an element sentence of a complex sentence and

   the complex sentence becomes null.[22]

For example, the following legal meta-rule sentence is to be applied to determine the second requirement of *r02*.

[r02-2]: A contract becomes null, if

   the contract is avoided.

In some cases, a legal element sentence alone becomes null independently of a legal complex sentence. For example, the following fundamental legal meta-rule sentence must be valid:

[mr4b]: A legal sentence 'B must do Z' becomes null at time T, if

   B performs the action Z at T.

4.4. The Union of Legal Meta Sentences and other Legal Meta Sentences

A legal meta rule sentence which determines if a legal sentence is valid is also to be proved as valid. The validity of «positive» legal meta-rule sentences is to be regulated by other legal meta-rule sentences. The union of positive legal meta-rule sentences and other legal meta-rule sentences to prove the validity of the

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[19]   In this rule sentence «may» can be replaced by «can» without changing its meaning.

[20]   HART (note 2), p. 81.

[21]   To execute a right means to issue a legal sentence which makes the relevant legal sentence legally valid or invalid.

[22]   The term «*become null*» is a technical term introduced here. Usually «is terminated» is used. These are synonyms.

former legal meta-rule sentences has the same logical structure as above clarified for the proof of the validity of legal object sentences.

The validity of fundamental legal meta-rule sentences is not regulated by other legal meta-rule sentences but declared by legal meta-fact sentences because they must be always valid. It should be represented, for examples, in the following fact sentences:

is_valid(r0,T). is_valid(r01,T). is_valid(3aa1,T). is_valid(3aa2,T). is_valid(r02,T).[23]

The external logical structure of the union of legal sentences which constitute the multi-layered or hierarchical structure of law can be clarified in this way.[24]

5. Concluding Remarks

In this paper, the author has clarified the internal logical structure of legal sentences in terms of legal rule and fact sentences, legal element and complex sentences, and legal object and meta-sentences. Based on the clarification, the author has clarified the external logical structure of legal sentences in terms of the union of these legal sentences. The author believes that this clarification of the logical structure of legal sentences provides the general basis for the logical systematization of law so that it will contribute to the construction of a total system of LKB and to approach the genuine science of law in future societies.

6. Acknowledgement

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[23] «*T*» is a variable so that it represents the relevant rule sentence is valid at any time.

[24] The author has once presented a small paper on this topic for AI and Law community. Cf. Yoshino, Hajime, The Systematization of Law in Terms of the Validity, in: Procedings of the Thirteenth International Conference on Artificial Intelligence and Law, ACM, 2011, pp. 121–125. A deep legal theoretical analysis and discussion with real examples should be provided for legal theory communities as a future task.