THE APPLICATION OF FAIR DIVISION SYSTEMS IN CASES INVOLVING THE JUDICIAL DIVISION OF ASSETS

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Abstract: *This paper presents the evolution and application of ODR systems in case involving the judicial division of assets. The CREA Project aims to introduce new mechanisms of dispute resolution in legal procedures for lawyers, mediators and judges with the objective to guarantee an equitable distribution between the parties involved in a dispute, and to reach an agreement. An application to a legal example illustrates such procedure.*

1. The evolution of the ODR systems

The aim of the Online Dispute Resolution is to resolve controversies in cases where the authority of the law is absent or inadequate. The first ODR systems had a tendency to imitate the dispute resolution channels of the Alternative Disputes Resolution (ADR) processes in order to offer valid solutions that could also work online. However, the main problems were that i) the users are unable to have face-to-face interactions, ii) ODR systems automatically record all the data of the dispute within their system iii) users rely on a computer for the solution of the dispute.

To date, there are many studies on ODR systems, as there are many systems designed to resolve disputes in this manner[1]. One of the most popular ODR systems is SquareTrade, used by the e-commerce site eBay to resolve disputes online. The well-known e-commerce site immediately attempted to put in place a system for the resolution of disputes, with the initial goal of reducing them, by introducing a user rating system. The e-commerce giant has, thus, created an online community of users, highlighting the good intentions of most of them and the need to communicate any problems within the community on a case by case basis[2]. SquareTrade, created by the University of Massachusetts, is a system that utilises two phases of assisted negotiation: the first

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1 We mention, in chronological order: S.J. Brams/A.D. Taylor, Fair-Division – From cake-cutting to dispute resolution. Cambridge University Press, NY 1996; J.ZELEZNIKOW/A.STRANIERI. Split up: an intelligent decision support system which provides advice upon property division following divorce. *International Journal of Law and Information Technology* 6.2 (1998): pp.190-213; E.Katsh/J.Rifkin Online dispute resolution: Resolving Conflicts in Cyberspaces, Wiley (2001); A.Lodder/E.Thiessen, The role of artificial intelligence in online dispute resolution. Workshop «Online Dispute Resolution» at the International Conference on AI and Law, Edinburgh 2003; A.R.LODDER/J.ZELEZNIKOW, Enhanced Dispute Resolution Through the Use of Information Technology, Cambridge University Press, 2010; M.Araszkiewicz/A.Lopatkiewicz/A.Zienkiewicz, The role of new information technologies in alternative resolution of divorce disputes. European Science Journal 2014.

2 V.G.Lindsy, The Strange Case of the Electronic Lover, Ms. Magazine (Oct. 1985).

uses online forms to present claims and request exchanges, while the second one contemplates the intervention of a professional mediator, in the event that a solution should not be reached within the first phase. Another alternative online dispute resolution system is Smartsettle, developed by Ernest Thiessen[3]. The system is based on the blind bidding negotiation mechanism[4] and it is applicable in any situation where there are multiple decision makers, both individually and in groups. Smartsettle is a software based on Game Theory insights that lists parties' interests and assigns them numerical values, creating a set of points on which the parties could negotiate, providing the solution that can satisfy the needs of the actors involved[5]. The system is able to solve both single-issue two-party case and individual decision making. It is enhanced with Smartsettle Facilitator, which helps to manage more complex issues. Spliddit system, unlike the previously analysed ODR systems, exploits fair division mechanisms[6] based on mathematical algorithms. It was created on November 4th, 2014 by the scholars Ariel Procaccia and Jonathan Goldman and includes a series of applications for fair division, such as the Share rent, the divide Goods and the Assign Credit. Each application has a unique execution algorithm, to guarantee an equitable distribution between the parties. Spliddit is not-for-profit and intends to pursue the following objectives[7]: (i) providing easy access to methods of fair division in disputes, making the world more equitable and (ii) communicating the beauty and value of theoretical research in computer science, mathematics and economics to the public, from a different perspective.

2. CREA Project – Conflict resolution with equitative algorithms

The Conflict Resolution through Equitative Algorithms (CREA) project has received funding from the European Union's Horizon 2020 research and innovation programme for two years of work. The project has started in October 2017 and it is based on revisions and innovations on the Spliddit algorithm, and specific adaptations to the law domain.

CREA project focus on division procedures that are not only proportional but also «envy-free». This way, they produce allocations in which each participant believes he or she receives the largest portion of the goods being divided, or obtains his way on more issues, based on their subjective references. The methodology has been applied to the allocation of goods[8] in areas such as divorce and estate division.

- Divorce: the algorithm is able to guarantee the equitable division, evaluating the concrete interest of the ex-spouses with regards to the various assets, analyzing the following aspects: special role imposed by law, matrimonial regime, list of goods to be divided (for each tangible good, specify: the market value, can/cannot be shared, must be assigned to a specific ex-spouse), amount of money available for the division, list of the debts/liabilities;
- Estate division: the algorithm is able to guarantee the equitable division, evaluating the concrete interest of the heirs with regards to the various assets belonging to the estate, analysing the following aspects: parties involved (rules regarding specific parties – increase or decrease of share determined by law), list of assets available for division. (for each tangible good, specify: the market value, can/cannot be

[3] An engineer who sensed that the introduction of technology in a negotiation based on a «Getting To Yes Interest» system could increase the probability for the parties to reach satisfactory results.

[4] In blind bidding negotiations, suggestions are made visible to all parties and by the computer that acts as an intelligent agent, but the approvals of each party are kept hidden from the other party. The computer announces a deal only when the hidden approvals coincide.

[5] E.M.Thiessen/J.P.McMahon, Beyond Win-Win in Cyberspace, Ohio State Journal on Dispute Resolution (2000).

[6] The fair division mechanism consists in the division of one or more assets among the players of the whole N = {1, ..., n} with heterogeneous preferences.

[7] J.Goldman, A.D. Procaccia, Spliddit: unleashing fair division algorithms, in ACM SIGecom Exchanges, vol. 13, Issue 2 (2014), pp. 41-46.

[8] For a more advanced approach we refer to A.Bogomolnaia, H.Moulin, F. Sandomirskiy, E. Yanovskaya, Competitive division of a mixed manna, Econometrica, Vol. 85 (2017), 6, pp. 1847–1871.

shared, must be assigned to a specific heir / cannot be assigned to a heir), amount of money available for the division, list of the debts/liabilities.

If a procedure works well in these cases, it might be used in more complex disputes. The approach could help judges and lawyers to set the legal procedure not as a confrontation clash of the parties, but as a process aimed towards the consensual agreement. The judge and the lawyer will not be viewed as custodians of the right or true legal solution anymore, but only as an aid to the parties who become the authors of the solution.

2.1. Procedures for the CREA project

Point allocation methods that are the basis of many fair division procedures are, e.g. the Adjusted Winner procedure. Quoting the Wikipedia

> «Each player is given the list of goods and an equal number of points to distribute among them. He or she assigns a value to each good and submits it sealed to an arbiter.»

Dealing with the division of an asset, however, when parties allocate points to items to express their likes and dislikes, the market value of the disputed items involved cannot be ignored. This happens, because if no agreement between the parties is found, items can be sold to third parties, and the collected money shared between the parties. Also, experience suggests that people are more familiar with prices than they are with absolute numbers. We, thus, propose a special point allocation method in which:

- Parties assign prices to items instead of absolute numbers;
- Each party will assign a personal price to each item. These subjective prices usually differ (but not too much) from the market prices to reflect the personal preferences of the party.

To exemplify, consider two parties who must divide among themselves, a house that worths 100000 euros, a motorbike that worths 10000 euros and other items. The allocation of points by all parties should reflect the fact that the first item is worth ten times more than the second one. Of course, the evaluations are personal and, therefore, the ratio between the two evaluations does not need to be exactly equal to 10 but must be close to this value. Instead of assigning absolute points, we can attach prices, thus, it will be reasonable that the first party assigns 90000 (euros) to the house and 12000 (euros) to the motorbike, meaning that she is eager to receive the bike, less so to get the house, while the second party assigns 110000 to the house and 8000 to the motorbike, implying opposite preferences. Certainly, it is unreasonable to assign, say 55000 to both items. In what follows, we propose two different methods for the determination of these subjective prices and we will propose two different procedures to actually determine a solution for the allocation of the contested items.

2.2. Explanation of the procedures

- Goods to be divided are listed and parties involved, also called players or agents, are defined.
- Two different solutions to assign goods to players are proposed: In each solution, one or more goods may be split among several parties in different proportions (for instance, the solution may require that a house is divided among three heirs, with the first heir entitled to ½ of the house, the second heir ⅓, and the third heir 1⁄6).
- Each player is given a share of entitlement. Typically, if there are n players, the share will be 1/n for each one of them, but it could be different. For instance, shares could reflect the closeness of relatives to a deceased person, or the effort.

Evaluation of preferences. We consider two methods for making the parties involve express their preferences.

- **Method 1**
  - For each good, a market price is given;

- The sum of the market prices is computed. This is the budget available to each player in the following steps;
  - An equal budget for each player reflects the principle that all players should be treated equally. Only the share of entitlement could discriminate among players.
- Each player is asked to distribute the budget as bids over the unassigned goods. Each bid cannot be less than the lower bidding value, and the total value of bids cannot exceed the budget.
  - The idea is that the higher the bid, the more likely is for the agent to receive the good.
  - The bids should reflect the utility of the good for that player – remembering that the good can be sold to the market, if no one bids higher than the lower bidding bound.
- It is reasonable to assume that no bid can be lower than a threshold, which we will call the lower bidding bound. The rationale behind this restriction is that in case of very low bids, a good can be sold to the market. The lower bidding bound should be a low enough price to guarantee the selling of the good (or at least provide an extremely high probability for its selling). Below this price, an offer cannot be considered acceptable. Of course, the lower bidding value varies from item to item, but a reasonable general rule is to decrease the market price by a fixed percentage, say 20%. Each player will participate to the allocation procedure with his/her own bids over the goods.

– **Method 2**

- Each disputed good is valued at the market price;
- Each player evaluates how much he/she would like to receive each good. The evaluation can be attained through a «1 to 5 stars» marking system (as the rating of an Amazon product or that of a restaurant through tripadvisor). This evaluation does not regard the monetary value of the good. For instance, a player is involved in the allocation of a house worth 100000 euros and one Harley-Davidson motorbike worth 10000 euros. I know that the house is worth more, but I already own a beautiful house, and I know that managing a house is time and money consuming. On the other hand, I have always dreamt about riding that motorbike. I will give 2 stars to the house and 5 to the bike. A more refined rating system could be used: for instance, a range of 7 or 10 stars.
- The evaluation of the good by the player is given by the market value and is modified by a percentage, depending on the number of stars assigned by the player. A proposal could be:
  - 1 star = market value decreased by 20%
  - 2 star = market value decreased by 10%
  - 3 star = market value unchanged
  - 4 star = market value increased by 10%
  - 5 star = market value increased by 20%

Each player will participate to the allocation procedure with his/her own star-modified evaluations. Notice that assigning 5 stars to all items will not make you any better off than assigning 1 star to all of them. What counts is the profile: You raise the chance of getting the items you really want by assigning them a high mark, and by giving a low mark to those you are not too interested.

Finding a solution: The optimization problem is run, considering the share of entitlement of each player. For each player, the utility of a good is given by the players' bids (Method 1) or by the star-modified market prices (Method 2). In the end, the overall utility of a player will be given by the sum of the utilities of the goods received. If a player gets a fraction of the good, the utility will be weighted by this fraction. For instance, if, according to the proposed solution, I should receive ⅓ of a house which I valued 90000 euros, my utility will be 90000 *(⅓) = 30000 euros. How to find an optimal allocation of the goods? Research suggests that no single criterion outperforms the others. The recent literature shows that two criteria prevail:

- **Solution 1: The egalitarian equivalent allocation.** This criterion was introduced by Pazner and Schmeidler in 1978[9]. If agents are entitled to the same share, it makes sure that all agents receive goods (or parts of them) such that the sum of the goods' value according to his/her own bids is the same, and this value is as high as possible. In case of different entitlement quotas, equality is attained once values are weighted with the shares in order to attain equality. By construction, this solution is egalitarian. It turns out that this solution is also efficient: No other allocation, even a non-egalitarian one, can make all agents better off simultaneously. The allocation, however, may fail to verify other interesting properties. In fact, the allocation may cause one or more players to be envious of some other players.
- **Solution 2: The Competitive Equilibrium from Equal Income/Nash Allocation.** This solution traces back to the seminal work of J.F. Nash, but it is has been thoroughly re-examined in a very recent work. Suppose each agent is given the same budget (something similar to what happens in the present procedure, but the budget covers only the share to which the agent is entitled. For instance, if the whole asset is worth 600000 euros and there are 3 agents all equally entitled, each will have 200000 to spend). Each good is given a price and each agent spends the budget so to maximize his/her own satisfaction (or «utility» in the economic vocabulary) buying goods at the fixed price. If goods are bought such that a) each player, independently of the others, makes the best choice: given the budget, he/she buys goods that maximize his/her own satisfaction and b) all goods are sold with no overlaps (for instance two agents buying the same good in its entirety) and no leftovers (no good remains unsold and the market «clears»), the Competitive Equilibrium from Equal Income (CEEI) solution is reached. It can be shown that, in the given circumstances, such equilibrium always exists, and it is simple to compute. In the above cited references, it is shown that this allocation can be equivalently obtained as the solution of an optimization problem in which the sum of the logarithms of the players' utility is maximized. This objective function was introduced by J.F. Nash[10] in a bargaining context and thus takes the name of Nash bargaining solution. This solution also enjoys several properties (we refer to the above references for details).
  - It is efficient and envy-free,
  - it does not suffer from domination.
  - It is Resource Monotonic: more goods to divide should not be bad news for anyone.
  - It has Responsive Shares: if a player raises the bid on a certain good, he/she cannot end up with a smaller share of that good.
  - It is Independent from Lost Bids: if a player changes a bid on an unassigned good, this will have no consequences on the resulting allocation which will remain as it was before the change.

Adjusting the solution. The optimization problem returns a solution. Players receive the unsplit goods. For the goods that, according to the solution, must be split, negotiations should take place. A negotiation for each good should begin, and should involve only the players entitled to some fraction of the good. Players may decide:

- To manage and enjoy the good together. This could occur, for instance, with a house or a piece of land.
- To sell the good and divide the proceeds according to the shares specified in the solution.

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9 E.A. Pazner, D. Schmeidler, Egalitarian Equivalent Allocations: A New Concept of Economic Equity, Quarterly Journal of Economics, Vol. 92 (1978), 4, pp. 671-687.

10 J.F. Nash, The Bargaining Problem, Vol. 18 (1950), 2, pp. 155-162.

– To bargain so that one part receives the good in its entirety, and the others receive side payments. If possible, the party that gets the good should not recur to his/her own money, but the money already forming the estate should be used instead. In this way, instead of the classical solution where every player receives one *n*-th of the cash forming the estate, each player will receive a different amount of money, based on the outcome of the all the bargaining in which that player is involved.

3. An Application

Here, it is shown how dispute resolution system algorithm applies to a case of inheritance. offering fair division for conflict resolution by means of the Egalitarian and the CEEI/Nash equilibrium.

Example of a legal case:[11] During his life, X was the owner of a land plot in Zadar with a building and garden (180m^2) with three flats: one on the ground floor (90m^2, 180 000 Euros), one on the first floor (60m^2, 120 000 Euros) and one on the second floor with a wonderful view of the shore and beach (60m^2, 130 000 Euros). All flats were condominiums and were rented out. He also owned another land plot in Zagreb with a building with three flats; one on the ground floor (55m^2, where his son A had a mechanic's workshop, 77000 euros not including equipment), one on the first floor (55m^2, where X lived, 80 000 Euros) and one on the second floor (45m^2, but needs full renovation, 45000 Euros). This second building was not a condominium. After the death of person X, he is succeeded by his sons, A, B and C. A is most interested in the ground floor because he operates a mechanic's workshop which is crucial for his livelihood. He wouldn't mind getting another apartment either in Zagreb or in Zadar. B already had a house, so he was interested in the house in Zadar. He wants two flats, the one on the first floor but especially the one on the second floor (this is his mayor priority). C has a tourist agency and he wants all the flats in Zadar.

The example is solved by considering Method 1. A lower bound for the items' prices is fixed in order to represent the minimal offer that each heir is allowed to present. The difference between the market price and the lower bound represents the amount of money that each heir is asked to allocate according to his or her preferences. Finally, the Egalitarian and Nash algorithms apply in order to fairly divide the items among the heirs.

Fix a lower bound for the bid: 20%. Then, the prices of the six apartments are:

		Zadar			Zagreb	
	GF	1	2	GF	1	2
Market price	180.000	120.000	130.000	77.000	80.000	45.000
Lower Bound	144.000	96.000	104.000	61.600	64.000	36.000

The Lower Bounds represent the minimum prices that each heir has to respect for the apartments in the Inheritance.

– Let the heirs offer the amount of money they believe the most adequate for each item in the patrimony. The maximum that each heir may allocate when expressing his or her preferences is equal to the maximum value of the sum of all the items in the patrimony, that is euros 632.000.

Note that no offer can be below the minimum prices expressed by the lower bound.

[11] We mention here one of the 36 cases brought to us by the Legal Workgroup of the CREA project.

- For example, A may be willing to offer euros 100.000 for the ground floor in Zagreb, as he claims the apartment is crucial for his livelihood, and equally redistribute the remaining amount among the other apartments. B may translate his special preference for the 2nd floor of the building in Zadar with an offer of 20% more and may offer 10% more for the 1st floor of the same building, he is not interested at all in the building in Zagreb. Similarly, C may equi-distribute his preferences in the apartments in Zadar.

The allocation of the total offer will be as follows.

		Zadar			Zagreb		Sum
	GF	1	2	GF	1	2	
A	172.000	115.000	125.000	100.000	77.000	43.000	632000
B	182.400	132.000	156.000	61.600	64.000	36.000	632000
C	200.000	130.000	140.400	61.600	64.000	36.000	632000

The allocation proposed by Solution 1 is the following:

- A gets all the flats in Zagreb and a 13% share of the first flat in Zadar
- B gets the second floor in Zadar and a 60% share of the first flat in Zadar
- C gets the ground floor in Zadar and a 27% share of the first flat in Zadar

If the heirs want to separate their ways, B could buy A's and C's shares of the first floor in Zadar. Alternatively, the flat could be sold, and the money shared in the given proportions.

The allocation proposed by Solution 2 is the following

- A gets all the flats in Zagreb
- B gets the second floor in Zadar and a 68% share of the first flat in Zadar
- C gets the ground floor in Zadar and a 32% share of the first flat in Zadar

Here, the bargaining following the algorithm looks simpler.

A thorough analysis of the manipulability (or gaming) of the system by those who know the algorithm and the preferences of the other parties involved in the division is currently under investigation.

4. References

M.Araszkiewicz, A.Lopatkiewicz, A.Zienkiewicz, The role of new information technologies in alternative resolution of divorce disputes. European Science Journal 2014.

A.Bogomolnaia, H.Moulin, F. Sandomirskiy, E. Yanovskaya, Competitive division of a mixed manna, Econometrica, Vol. 85 (2017), 6, pp. 1847–1871.

S.J. Brams, A.D. Taylor, Fair-Division – From cake-cutting to dispute resolution. Cambridge University Press, NY 1996.

C. Corradi, V. Corradi, Nozione giuridica di divisione e fair division: alcune osservazioni, in «Rendiconti per gli studi economici quantitativi», 2005, 1, pp. 149 – 156.

J.Goldman, A.D. Procaccia, Spliddit: unleashing fair division algorithms, in ACM SIGecom Exchanges, vol. 13, Issue 2 (2014), pp. 41-46.

E.Katsh, J.Rifkin Online dispute resolution: Resolving Conflicts in Cyberspaces, Wiley (2001).

V.G.Lindsy, The Strange Case of the Electronic Lover, Ms. Magazine (Oct. 1985).

A.Lodder, E.Thiessen, The role of artificial intelligence in online dispute resolution. Workshop «Online Dispute Resolution» at the International Conference on AI and Law, Edinburgh 2003.

A.R.Lodder/J.Zeleznikow, Enhanced Dispute Resolution Through the Use of Information Technology, Cambridge University Press, 2010;

J.F. Nash, The Bargaining Problem, Vol. 18 (1950), 2, pp. 155-162.

E.A. Pazner, D. Schmeidler, Egalitarian Equivalent Allocations: A New Concept of Economic Equity, Quarterly Journal of Economics, Vol. 92 (1978), 4, pp. 671-687.

E.M.Thiessen, J.P.Mcmahon, Beyond Win-Win in Cyberspace, Ohio State Journal on Dispute Resolution (2000).

J.Zeleznikowia/A.Stranieri. Split up: an intelligent decision support system which provides advice upon property division following divorce. International Journal of Law and Information Technology 6.2 (1998): pp.190-213

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