

A NOVEL UNDERSTANDING OF LEGAL SYLLOGISM AS A STARTING POINT FOR BETTER LEGAL SYMBOLIC AI SYSTEMS

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Abstract: *In the two most relevant categories of legal systems, the codified law systems and the case law systems, legal argumentation and legal reasoning are at the heart of legal theory and practice. Here we show that the concept of legal syllogism can be used as a starting point in both legal systems for new computational models of legal reasoning. We argue that legal syllogisms are not merely analytic logical inference rules but involve ampliative analogical inference. We aim to operationalize our account in terms of Context Graphs, i.e. internally consistent logical theories linked through rigorously defined analogical relations called views and argumentative relations such as attack and support.*

1. Introduction

A classical, axiomatic view of the law conceives of the law as a logical system. Inspired by this a naïve computer scientist might roughly view the law as an expert system: general legal rules as laid down in the law (codified or not) constitute its TBox; concrete facts of a case its ABox; a collection of legal syllogisms the logical rules of its inference system.

Obviously, such a system would require an immense formalization effort to capture the complete “axiom system” and an even larger ontology building effort. In addition such an effort would be fraught with difficulties such as the (often intentional) ambiguity of much of the law.¹ However, even if these difficulties could be overcome the resulting concept of legal theory and the corresponding implementation would face at least three severe shortcomings:

- Jurists and an expert systems alike have to address that laws may sometimes be in conflict with each other rendering a naïve axiom system inconsistent.
- They have to contend with the problem that the present case is uncharted territory and not yet known to the law – i.e. there is a gap in the axiomatic system.
- Finally, they have to assign the concrete present case facts to applicable abstract norms requiring every time the bridging of the divide between the finite abstract knowledge base and the infinite attributes of any individual case. of reality.

Jurists tackle the first problem by the method of interpretation to avoid such conflicts. Modern robust logics such as defeasible logics or argumentation systems can mimic this solution to an extent. These allow the

¹ GRÄWE, Die Entstehung der Rechtsinformatik, Wissenschaftsgeschichtliche und -theoretische Analyse einer Querschnittsdisziplin, 2011, p. 230ff.

derivations (arguments) computed by the inference system from axioms and syllogisms to be supported or defeated by other derivations. This defeat process is then resolved in an argumentative process which only the best defended arguments survive.

The second problem is addressed in the method of law development (*Rechtsfortbildung*) in the codified law system and distinguishing in the case law system, respectively. The third problem by the method of subsumption (in the specific sense). Every case that a judge has to adjudicate will vary in some factual details from ones that have previously been decided. In each case the subsumption task the judge faces is thus new in the strict sense: none of the subsumption rules in the knowledge base will be exactly fitting. Yet handling such new subsumptions is routine for any law practitioner.

However, for (at least symbolic) AI the second and third problem have proven much more hardy as they would require an infinite knowledge base that contains assignment rules for every possible eventuality. Obviously, such a knowledge base is impossible. In its absence, the only remedy is an inference system that is not merely analytic but ampliative (or synthetic in Kantian diction) in that it is able to create new subsumption rules and even legal rules as needed.

We will argue in the following that this ampliative aspect of legal reasoning is achieved by means of analogical reasoning. We want to defend the position that legal reasoning is in its entirety based on what is called legal syllogism. However, this notion of syllogism cannot be merely understood in terms of purely logical rules. Instead syllogisms must be conceived of as meta-logical operations that handle ampliative instantiation as well as analytic inference. This immediately poses the question how such reasoning can be implemented by AI. In recent years the boom in subsymbolic approaches such as deep learning with representation models such as word embeddings and architectures such as transformers has increased hopes to fruitfully apply such methods in the legal domain. However, in practice there are difficulties that limit the immediate applicability of these methods in the law.² In their present form they rely on retrospection based on large amounts of data. This is insufficient in a domain in which any amount of old data can become irrelevant due to a single court decision and in which whole areas are often defined by only few landmark cases. Thus subsymbolic methods presently do not provide us with the kind of ampliative inference we need. Neither do they provide us with what is likely the most crucial tool jurists use for such reasoning: legal arguments. Cases are not to be decided according to the number of factors in favor of one side or the other. This principle does not stop to apply if these numbers are aggregated into a statistical confidence measure. Instead the internal logic of the individual arguments and precedents is indispensable when assessing their strength and relevance.

This does not mean that there is no place for subsymbolic methods in legal AI. There is reason to hope that its strength in finding patterns in data is exactly what is needed to generate candidate analogies (in subsumption in a specific sense, *Rechtsfortbildung* and extensive interpretation/distinguishing). However, this can only succeed within a symbolic framework that provides both the necessary high precision structure of legal knowledge as well as *certificates* that ensure that every candidate analogy is warranted by a correct legal argument.

In the following we want to show what the concept of legal syllogism in the aforementioned understanding is about and why this can be a starting point for the representation of legal argumentation and (ampliative) legal reasoning with logic.³ We will argue that legal syllogism so understood can be used to represent the most important concepts of legal thinking in both most relevant legal systems, the codified law systems (e.g.

² BENCH-CAPON. The Need for Good Old fashioned AI and Law. In W. Hotzendorfer, C. Tschohl and F. Kummer (Hrsg.) *International Trends in Legal Informatics: A Festschrift for Erich Schweighofer*. Editions Weblaw: Bern. Pages 23–36, 2020.

³ We explore how such a representation could look like in our other contribution in this issue: “Context Graphs for Ampliative Legal Reasoning and Argumentation”.

of Germany⁴, Austria⁵, France, Italy, the European Union⁶, etc.) and the case law systems (e.g. of USA, UK⁷). This is the legal-theoretical basis for a new computational model of legal reasoning in terms of operations on *Context Graphs*, i.e. internally consistent logical theories linked through rigorously defined analogical relations called views and argumentative relations such as attack and support.⁸

2 Legal Theory: Foundations and the Role of Syllogism

The most important questions in legal theory are about the structure of legal thinking and its methods⁹ and about how to discover these methods¹⁰. Legal scholars discuss, for example, whether it is possible to make legal decisions based on logical, i.e., axiomatic-deductive arguments: Can judgments in legal cases be logically deduced from the law? Or can legal cases only be decided with associative and intuitive methods such as the distinction by cases?¹¹ It is an important issue to prescribe a theory of correct legal reasoning and legal argumentation i.e. to root the binding of the judge to the law in the principles of democracy and the rule of law¹². It is also of interest how these questions are viewed and answered in different legal systems, such as codified law systems and case law systems.¹³ Even though there is a huge literature in legal theory regarding the methods of jurisprudence¹⁴, there is still no consensus of how legal reasoning and legal argumentation actually works in detail nor how it should in fact work.

However, in either law system, within the process of interpretation and application of law there is at least one main issue which is clear enough to be represented by computer-science-methods: legal practitioners in either system structurally combine some legal rule from statute law or the “rule of a case” from case law with the legally important facts of the present case.¹⁵ There are various formal models on such application/subsuming of facts and rules, some of which use non-classical logic and argumentation theory¹⁶. Although it is debated how

⁴ ENGISCH, Einführung in das juristische Denken, 1997; CANARIS and LARENZ, Methodenlehre der Rechtswissenschaft, 1995; H.-C. RÖHL und K. F. RÖHL, Allgemeine Rechtslehre, 2008; ZIPPELIUS, Juristische Methodenlehre, 2012; ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009; *id.* Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014.

⁵ BYDLINSKI, Juristische Methodenlehre und Rechtsbegriff, 1991.

⁶ ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009.

⁷ ASHLEY, Artificial Intelligence and legal Analytics, 2017; BURNHAM, Introduction to the Law and Legal System of the United States, 2016; KEENNAN, English Law, 1993; BLUMENWITZ, Einführung in das anglo-amerikanische Recht, 2003; HART, The Concept of Law, 2012; MACCORMICK, Legal Reasoning and Legal Theory, 1978; FRIEDMAN and HAYDEN, American Law, An Introduction, 2017; MCLEOD, Legal Method, 2019.

⁸ RAPP, ADRIAN und KOHLHASE, Context Graphs for Legal Reasoning and Argumentation, Proceedings of the Third International Workshop on Systems and Algorithms for Formal Argumentation co-located with the 8th International Conference on Computational Models of Argument (COMMA), 2020, p. 56–67.

⁹ ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 590 ff.

¹⁰ *IBID.*, p. 590 ff.

¹¹ ASHLEY, Artificial Intelligence and legal Analytics, 2017, p. 54–55, 127–129; ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 92, 283, 472 ff., 538, 597 f., 880 ff.

¹² MACCORMICK, Legal Reasoning and Legal Theory, 1978; ZIPPELIUS, Juristische Methodenlehre, 2012; ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 83 ff., 879.

¹³ BLUMENWITZ, Einführung in das anglo-amerikanische Recht, 2003, p. 3 ff.

¹⁴ Cf. e.g. ASHLEY, Artificial Intelligence and legal Analytics, 2017; BURNHAM, Introduction to the Law and Legal System of the United States, 2016; KEENNAN, English Law, 1993; BLUMENWITZ, Einführung in das anglo-amerikanische Recht, 2003; HART, The Concept of Law, 2012; MACCORMICK, Legal Reasoning and Legal Theory, 1978; BYDLINSKI, Juristische Methodenlehre und Rechtsbegriff, 1991; ENGISCH, Einführung in das juristische Denken, 1997; CANARIS und LARENZ, Methodenlehre der Rechtswissenschaft, 1995; H.-C. RÖHL und K. F. RÖHL, Allgemeine Rechtslehre, 2008; ZIPPELIUS, Juristische Methodenlehre, 2012; MCLEOD, Legal Method, 2019.

¹⁵ ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 879. ASHLEY.

¹⁶ ASHLEY, Artificial Intelligence and legal Analytics, 2017, p. 127.

the rule of a case and the legally relevant facts are inferred, there is broad consensus in the legal literature that this structural combination can be carried out by means of syllogism, i.e., axiomatic-deductive arguments¹⁷.

2.1 Legal Reasoning and Argumentation – Legal Syllogism

Independent of the concrete jurisdiction, there are two categories of legal knowledge that legal methods have to deal with. On the one hand there are legal rules, written by the legislator¹⁸ and on the other hand precedents¹⁹. Knowledge-based applications usually represent these via rule-based-reasoning²⁰ or case-based-reasoning²¹. Some distinguish further between rule and analogy application.²² The most important methods of legal reasoning and legal argumentation within these two systems and categories in the English jurisdiction and language²³ (in comparison to the German jurisdiction and language²⁴) are:

- mere interpretation (in German jurisdiction: *Auslegung*),
- extensive interpretation (statute law) or distinguishing (case law) (both in German jurisdiction: *Analogie/ Rechtsfortbildung*),
- application of Law (in German jurisdiction: *Subsumption*).

Despite the fact that these methods are usually differentiated, there seems to be broad consensus with regard to the main task of legal reasoning and legal argumentation. This is to construe or interpret legal texts in a certain way to determine their legally correct meaning.²⁵ Ultimately, legal practitioners are to ensure that the meaning of the text given by the prejudice of a leading decision or by the legislator is decisive for the judicial decision and not the decision maker's personal opinion²⁶. We follow the approach that all of these methods, even the method of distinguishing, can be formalized with a legal syllogism²⁷, which will be presented in the following. The reasoning scheme of the legal syllogism is as follows:

1. Major Premise ("Obersatz"): A is defined in terms of B_1, \dots, B_n ;
2. Minor Premise ("Untersatz"): C satisfies B_1, \dots, B_n ;
3. Conclusion ("Schlussatz"): C is an instance of A.

¹⁷ MACCORMICK, *Legal Reasoning and Legal Theory*, 1978, p. 19 ff.; BYDLINSKI, *Juristische Methodenlehre und Rechtsbegriff*, 1991, p. 91 ff.; ENGISCH, *Einführung in das juristische Denken*, 1997, p. 89; CANARIS und LARENZ, *Methodenlehre der Rechtswissenschaft*, 1995, p. 273; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 123 ff.; ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 79 ff.; WEINBERGER, *Rechtslogik*, 1989, p. 145 ff.; JOERDEN, *Logik im Recht*, 2009; MCLEOD, *Legal Method*, 2019, p. 10 ff. For a discussion see e.g.: MACCORMICK, *Legal Reasoning and Legal Theory*, 1978, S. 19 ff., 229 ff.; HART, *The Concept of Law*, 2012, p. 124 ff. or ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 777 ff.

¹⁸ BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 63 ff.; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 519 ff.; MCLEOD, *Legal Method*, 2019, p. 225 ff.;

¹⁹ BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 11 ff.; MCLEOD, *Legal Method*, 2019, p. 123 ff.

²⁰ ASHLEY, *Artificial Intelligence and legal Analytics*, 2017, p. 38 ff.

²¹ *IBID.*, p. 71 ff.

²² ATKINSON and BENCH-CAPON, "Reasoning with Legal Cases: Analogy or Rule Application?". In: *Proceeding of the seventeenth International Conference on Artificial Intelligence and Law*, 2019, pp. 12–21.

²³ BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 36 ff., 71 ff.

²⁴ ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 35 ff., 52 ff., 71 ff.

²⁵ See ASHLEY, *Artificial Intelligence and legal Analytics*, 2017, p. 52 regarding especially the process of interpretation of statutory law; but also see the pertinent German literature: BYDLINSKI, *Juristische Methodenlehre und Rechtsbegriff*, 1991, p. 428 ff.; ENGISCH, *Einführung in das juristische Denken*, 1997, p. 159 ff.; CANARIS and LARENZ, *Methodenlehre der Rechtswissenschaft*, 1995, p. 141; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 24; ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 15 ff., 37 ff.

²⁶ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 35.

²⁷ MACCORMICK, *Legal Reasoning and Legal Theory*, 1978, p. 19 ff.; BYDLINSKI, *Juristische Methodenlehre und Rechtsbegriff*, 1991, p. 41 ff.; ENGISCH, *Einführung in das juristische Denken*, 1997, p. 89; CANARIS and LARENZ, *Methodenlehre der Rechtswissenschaft*, 1995, p. 273; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 123 ff.; ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 79 ff.; WEINBERGER, *Rechtslogik*, 1989 p. 145 ff.; JOERDEN, *Logik im Recht*, 2009, p. 336 ff.; MCLEOD, *Legal Method*, 2019, p. 10 ff.; KLUG, *Juristische Logik*, 1966, p. 79, 120 ff.

This scheme is used in law application/subsumption to arrive at legal decisions. The Major Premise/Obersatz is obtained either directly from the letter of the law or through the methods of mere interpretation, extensive interpretation and distinguishing. In the naïve expert system it would be called from the TBox. The Minor Premise/Untersatz results from fact finding which itself applies subsumption to bring a given fact C under legal conditions B_1, \dots, B_n . In the naïve expert system the minor premise is a subsumption rule to be found in the ABox. We argue that instead the syllogism scheme involves analogical reasoning: in the mere interpretation case, the use of analogy is largely analytic: it does not generate any new information. In subsumption in a specific sense, *Rechtsfortbildung* and extensive interpretation/distinguishing the use of analogy is ampliative: a new subsumption rule is developed through an argument based on the structural similarity of the (abstract) domain defined in the major premise and the (concrete) domain described in the minor premise.

2.2 Legal Interpretation of codified law and statutes (Auslegung)

The interpretation of codified law and statutes (Auslegung) involves logical deduction. The following argument types are distinguished²⁸:

- Linguistic arguments; (In German: Wortlautargument).
- Systemic arguments; (In German: Systematisches Auslegungsargument).
- Teleological arguments from purpose; (In German: Teleologisches Auslegungsargument).
- Transcategorical arguments from intention; (In German: Subjektiv-teleologisches oder auch historisches Auslegungsargument).

All of these arguments can be understood as syllogisms where sets of narrower concepts and words are deduced from sets of broader concepts and words²⁹. Definition in the major premise and satisfaction in the minor premise are defined logically: in the example, the concept “mammal” is given a (partial, sufficient) definiens in terms of the concept “dog” (formally: $\forall x.dog(x) \rightarrow mammal(x)$) and satisfaction of that definiens by the concept mastiff corresponds to a logical formula that allows to establish the truth conditions of the definiens (here: $\forall x.mastiff(x) \rightarrow dog(x)$).

Every dog is a mammal.
 Every mastiff is a dog.
 So every mastiff is a mammal.

2.3 Creation/Development of new law

There is probably no standardized definition which shows how analogous application of legal rules differs between case law and statute law systems. We attempt one here:

Analogy/Rechtsfortbildung: While interpretative and subsumptive reasoning (ampliative or not) in general can already be seen belonging into the sphere of analogy, parts of the legal literature, based on German law, define “Rechtsfortbildung durch Analogieschlüsse” as the area lying beyond the possible meaning of the letter of the law. There are criteria for applying a legal rule to a case which is not mentioned in the rule’s wording: If a fair and legally satisfying solution cannot be achieved by applying legal norms through mere interpretation, then the legal norms themselves seem to be in need of amendment. A judge must therefore find

²⁸ ASHLEY, *Artificial Intelligence and legal Analytics*, 2017, p. 53; BYDLINSKI, *Juristische Methodenlehre und Rechtsbegriff*, 1991, p. 428 ff.; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 613 ff.; ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 35 ff; MCLEOD, *Legal Method*, 2019, p. 246 ff.

²⁹ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 821ff, 863ff, 865ff, 868ff, 887ff .

a (practical) solution for a case that requires regulation.³⁰ As Ulrich Klug already demonstrated in the 1960's with the aid of modern logic, ultimately the difference between permissible and non-permissible *Rechtsfortbildung* i.e. analogy is a question of definition, which is still in the area of the similar³¹. Only if the definition is at hand can the analogy be logically concluded. Klug refers to this as “similarity circle” (“Ähnlichkeitskreis”³²).

The scheme on the right serves for clarification. Satisfaction in the minor premise is now defined in terms of a similarity circle. Here doglike is the similarity circle, by which an exact conclusion is made possible³³. Once the similarity circle's definition has been established it can be logically derived whether the analogy is permissible or not. Such interpretation may be ampliative or not – in this way capturing the concept of *Rechtsfortbildung* in the syllogism scheme.

Every doglike is a mammal.
Cats are doglike.
So every cat is a mammal.

Distinguishing: In case law systems the method of distinguishing and development of law is very sleek and flexible. In principle every case differs from other cases³⁴. Thus only that is decided which was brought before the court by the parties. This opens the possibility of distinguishing.

Regardless of its instance every court can independently review every previous verdict – even those of the supreme court – on its prejudicial power: for in the present case legally significant differences to the precedent may appear. On the one hand this differentiation of cases limits the ability to subsume cases under a norm. On the other hand new legal rules can be determined seeing that the law itself is set by the distinguishing decision of a previous instance³⁵. Distinguishing, like the “dictum”, is determined by the “ratio decidendi” of the precedent and sets the mental crux for the further development of the case. Frequently arguments to determine new legal rules arise from the subtle differences between cases³⁶: e.g. it could be objected that the *ratio decidendi* of prejudice in actuality was very different than argued by the opposing side; furthermore, parties may claim that the circumstances which underly the prejudice are different from the present case in legally relevant ways³⁷. If there are no factual differences between two cases the court “follows the previous verdict”; if there are factual differences, which however don't seem to be legally significant and seem to suggest an equal treatment of the cases the court “applies the precedent”³⁸. In conjunction with the knowledge and analysis of further cases a distinguished case may become a “leading case” regulating the formerly unlegislated space. In this way law is developed from a still vague “ratio decidendi” to the established and proven “rule of the case”³⁹. This occurs by a practitioners' recognition that one would have to derive different legal outcomes from the previous case law⁴⁰. Finally it can be stated, that the method of distinguishing can also be represented

³⁰ BYDLINSKI, *Juristische Methodenlehre und Rechtsbegriff*, 1991, p. 472 ff.; ENGISCH, *Einführung in das juristische Denken*, 1997, p. 235 ff.; CANARIS and LARENZ, *Methodenlehre der Rechtswissenschaft*, 1995, p. 187 ff.; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 633 ff.; ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 52 ff.

³¹ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 913 ff.

³² KLUG, *Juristische Logik*, 1966, p. 79, 120 ff.

³³ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 902 ff.

³⁴ BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 58.

³⁵ ASHLEY, *Artificial Intelligence and legal Analytics*, 2017, p. 71 ff.; BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 58; MACCORMICK, *Legal Reasoning and Legal Theory*, 1978, p. 219 ff.; McLEOD, *Legal Method*, 2019, p. 123 ff.

³⁶ ASHLEY, *Artificial Intelligence and legal Analytics*, 2017, p. 83 ff. with Hypo; BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 58; MACCORMICK, *Legal Reasoning and Legal Theory*, 1978, p. 224 ff.

³⁷ BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 59.

³⁸ IBID.

³⁹ ASHLEY, *Artificial Intelligence and legal Analytics*, 2017, p. 10 ff.; BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 61.

⁴⁰ BLUMENWITZ, *Einführung in das anglo-amerikanische Recht*, 2003, p. 61.

with a logical syllogism: the logical structure of the syllogism functions just as well for the comparison of cases with cases as for the comparison of law with cases.⁴¹

2.4 Limitations of common legal theory

So, as shown above, all relevant legal methods, even the method of analogical thinking and distinguishing, can be formalized by way of legal syllogism. With the classical logical operation of syllogism premises can be related to conclude whether law can be applied for a case or not⁴². Only the things represented by the premises which are related, subsumed or assigned with this syllogism differ according to the specific legal method, outlined above:

- cases assigned to law (in a codified law system cases assigned to norms/statutes made by the legislator; Application/Subsumption “in the general sense”)⁴³,
- concrete facts assigned to concrete words of law (in Germany: Subsumption “in the specific sense”)⁴⁴,
- narrower concepts/words assigned to broader concepts/words (Interpretation/Auslegung)⁴⁵,
- present case assigned to precedents/leading cases/sets of similar cases (Distinguishing/Rechtsfortbildung)⁴⁶.

The shown models of common legal theory can help in formalizing and criticizing legal reasoning and legal argumentation within a system of syllogisms on which an individual legal decision is based. However, as opposed to the classical logics this system of syllogisms involves (non-)ampliative analogical reasoning as well as other sources of defeasibility such as burden of proof and or default reasoning. Although logical and computational models for these aspects exist, they have not yet entered common legal theory and practice. In addition, in common legal theory it is still an open question how to formalize the argumentative questioning of the concrete framing in itself.⁴⁷ The definition of a modern legal theory, which could give the criteria for the testing of legal argumentation in syllogistic systems and questioning of these syllogisms themselves remains a challenge in jurisprudence.

3. Conclusion: Legal Theory and Legal-Tech

We have presented that syllogism can be fruitfully understood in an analogical, ampliative manner to model subsumption in a specific sense, *Rechtsfortbildung*, extensive interpretation and distinguishing. No machine can currently simulate such sophisticated legal thinking.⁴⁸ On the theoretical side common theory of legal methods cannot serve as a model for such machines if it maintains the underlying conception that words/symbols, reality and structure are congruent so that words would have a (concretely) determinable meaning.⁴⁹

⁴¹ E.g. ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 58 ff. compares distinguishing with German legal methods and outlines the similarity being the “Methode des typisierenden Fallvergleichs”.

⁴² MACCORMICK, *Legal Reasoning and Legal Theory*, 1978, p. 19 ff.; BYDLINSKI, *Juristische Methodenlehre und Rechtsbegriff*, 1991, p. 41 ff.; ENGISCH, *Einführung in das juristische Denken*, 1997, p. 89; CANARIS and LARENZ, *Methodenlehre der Rechtswissenschaft*, 1995, p. 273; H.-C. RÖHL and K. F. RÖHL, *Allgemeine Rechtslehre*, 2008, p. 123 ff.; ZIPPELIUS, *Juristische Methodenlehre*, 2012, p. 79 ff.; WEINBERGER, *Rechtslogik*, 1989, p. 145 ff.; JOERDEN, *Logik im Recht*, 2009, p. 336 ff.; McLEOD, *Legal Method*, 2019, p. 10 ff.

⁴³ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 778 ff.

⁴⁴ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 781 ff.

⁴⁵ *IBID.*, p. 887 ff.

⁴⁶ *IBID.*, p. 884 ff.

⁴⁷ ADRIAN, *Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre*, 2009, p. 235, 762 f., 847, 952; ADRIAN, *Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen*, 2014, p. 77, 86, 120, 117; ADRIAN, “Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie”, p. 103 f., In: *Rechtstheorie* 1, 2017, p. 77 ff.

⁴⁸ ADRIAN, “Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie”, p. 103 f., In: *RECHTSTHEORIE* 1, 2017, p. 77 ff.

⁴⁹ *ID.*, *Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen*, 2014, p. 65 ff.; *ID.* “Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie”, p. 82 ff., In: *RECHTSTHEORIE* 1, 2017, p. 77 ff.; *ID.*, “Juristische Methodenlehre

The meaning of a word/symbol can only be given by its usage in a specific context. Since LUDWIG WITTGENSTEIN's later writings and the "linguistic turn"⁵⁰ in the philosophy of language we have known that the text itself cannot contain any semantic meaning. The semantic meaning is rather only "construed" by the reader of the text.⁵¹

However, this is not the only problem: Legal thinking according to the common theory of legal methods seems to be structurally based on a self-referential paradox: Judges have to interpret the law first to then know how to interpret that law.⁵² Furthermore, it can be seen that all logical conclusions in classical legal thinking end up in an "infinite regress"⁵³ or in a "vicious circle"⁵⁴. Usage and context of words are therefore not (concretely) determinable but rather only given within a reference frame. Such a frame has to be agreed upon within an argumentative dialogue as itself is not determinable by being pre-established.⁵⁵ Thus modern models of legal thinking can only be reasonably criticized within an agreed upon framing. At the same time the agreed upon framing can in itself be argumentatively questioned.⁵⁶ Hence critical argumentation must and should be self-referring.⁵⁷ The testing of legal argumentation in systems of a modern legal theory of legal syllogisms and the questioning of the syllogisms themselves therefore become complex tasks. Hence, we proposed in previous works to model the argumentation within a chosen frame as well as the argumentative escape from that chosen frame with the methods of structural science. From the standpoint of scientific methodology the use of structural sciences opens to jurisprudence the world of higher complexity and predictability⁵⁸ – considering, however, the principle limitations of structural models.⁵⁹

On the technical side, the approach to syllogism we presented suggests the extension of the naïve expert system with an ampliative analogical inference engine. The required symbolic representation frameworks for such an inference system can be modelled through context graphs within the OMDoc/MMT language and system⁶⁰ ⁶¹. They enable the representation of both logical object languages and their inference rules as well as meta-logical relations. Specifically, "contexts" model possible theories, belief sets or worlds as consistent lists of (logical) declarations and "views" map the declarations of one context to the expressions (formed from

– Ein Vorbild für verantwortungsvolle Digitalisierung". In: In Verantwortungsbewusste Digitalisierung, Tagungsband des 23. Internationalen Rechtsinformatik Symposiums IRIS, 2020.

⁵⁰ ID., Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 65 ff.; ID., "Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie", p. 88 f., In: Rechtstheorie 1, 2017, p. 77 ff.

⁵¹ ID., Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 77 ff.; ID., "Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie", p. 93 f., In: Rechtstheorie 1, 2017, p. 77 ff.

⁵² ID., Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 762, footnote 643; ID., Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 31 ff., p 99 ff.

⁵³ See e.g. ID., Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 109, 121, 128.

⁵⁴ ALBERT, Traktat über kritische Vernunft, 1991, p. 13 ff., in particular p. 15 and ADRIAN Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 27, 32, 109, 121, 125, 128, ID. "Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie", p. 103 f., In: Rechtstheorie 1, 2017, p. 77 ff.

⁵⁵ ID., Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 946 ff.; ID., Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 31 f., 114 f.

⁵⁶ WOHLRAPP, Der Begriff des Arguments, 2009, p. 436 ff.

⁵⁷ ADRIAN, Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 235, 762 f, 847, 952; ID. Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 77, 86, 120, 117; ID. "Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie", p. 103 f., In: Rechtstheorie 1, 2017, p. 77 ff.

⁵⁸ ID. Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 111 ff., 126 f.; ID. "Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie", p. 82 ff., In: Rechtstheorie 1, 2017, p. 77 ff.

⁵⁹ ID., Grundprobleme einer juristischen (gemeinschaftsrechtlichen) Methodenlehre, 2009, p. 715 ff.; ID., Grundzüge einer allgemeinen Wissenschaftstheorie auch für Juristen, 2014, p. 49 ff., p. 98 ff.; ID. "Der Richterautomat ist möglich – Semantik ist nur eine Illusion in Rechtstheorie", p. 110 ff., In: Rechtstheorie 1, 2017, p. 77 ff.

⁶⁰ RABE and KOHLHASE, "A Scalable Module System". In: Information & Computation 0.230, 2013, pp. 1–54. url: <https://kwarc.info/frabe/Research/mmt.pdf>. Monographs.

⁶¹ RAPP, ADRIAN and KOHLHASE, Context Graphs for Legal Reasoning and Argumentation, Proceedings of the Third International Workshop on Systems and Algorithms for Formal Argumentation co-located with the 8th International Conference on Computational Models of Argument (COMMA), 2020, p. 56–67.

the declarations) of another. This mapping includes axioms which have to be mapped to proofs in the target theory. Such views can be used to model analogical relations: a theory is subsumed by another if instantiations of all of the latter's axioms can be proven in the former. Theories may stand in analogy with each other if they are instantiations of some common metatheory. Crucially, while contexts are internally consistent views also enable to model conflicts between them through mapping axioms to disproofs .

In this way context graphs allow to represent both the logical, analytic and analogical, ampliative aspects of legal syllogisms and therefore all the methods of legal argumentation and reasoning in codified and in case law systems discussed in this article. In addition, the framework even allows to take the above mentioned considerations into account at least partially by elevating from the object to the meta-language level.⁶² In addition, the framings discussed above can directly be modeled as views.⁶³ Thus even argumentative dialogues about the framing itself can be represented, which is important regarding the shown aspects of a modern legal theory.⁶⁴ In future work we hope to develop the ampliative analogical inference engine by leveraging context graph representations to automatically find pertinent subsumption candidates as well as attacks between them.

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⁶² In comparison e.g. ASHLEY, Artificial Intelligence and legal Analytics, 2017, p. 367 ff.; GRÄWE, Die Entstehung der Rechtsinformatik, Wissenschaftsgeschichtliche und -theoretische Analyse einer Querschnittsdisziplin, 2011, p. 230 ff.

⁶³ See KOHLHASE and M. KOHLHASE, “Spreadsheet Interaction with Frames: Exploring a Mathematical Practice”. In: MKM/Calculus Proceedings, 2009, pp. 341–356 for a discussion.

⁶⁴ If someone thought of an „analogy“, looking at the context graph diagrams for graphically and mathematically describing the interaction of legal arguments, with Richard Feynman's famous diagrams for describing the interaction of elementary physical particles, the first mentioned co-author of this article would be happy, as he had cheekily postulated a „quasi-quantum theory of legal language“ already in 2017 (ADRIAN, “Der Richterautomat ist möglich – Semantik ist nur eine Illusion”, p. 95 ff., In: Rechtstheorie 1, 2017, p. 77 ff.). See also BAEZ and STAY Physics, Topology, Logic and Computation: A Rosetta Stone, in New Structures for Physics, ed. Bob Coecke, Lecture Notes in Physics vol. 813, Springer, 2011, pp. 95–174.

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