CONTEXT GRAPHS FOR AMPLIATIVE ANALOGICAL LEGAL REASONING AND ARGUMENTATION

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Keywords: ***Knowledge Management, Argumentation, Logic, Theory Graph, Metalogical Frameworks, Subsumption, Analogy, Legal Reasoning***

Abstract: *Legal Reasoning involves both analytic/deductive and ampliative/analogical components. The latter are tightly connected to defeasibility in legal argumentation. Yet present rule-based models of defeasible legal inference are not ampliative: what is defeasibly derivable is solely determined by the contents of their knowledge base. We argue that a proper modelling of the analogical aspects of legal reasoning allows one to capture both much of its ampliative and much of its defeasible capabilities. Context graphs enable the representation of ampliative analogies as metalogical operations. We sketch algorithms acting on Context Graphs to implement analogical reasoning.*

1. Introduction

Analogical Reasoning is central to legal reasoning in at least two of the major categories of legal systems: the codified law systems and the case law systems. In the former, analogical reasoning features at least in law development (*Rechtsfortbildung)*, in the latter in the method of distinguishing. Arguably, however, already questions of mere interpretation and simple subsumtions of concrete facts under abstract legal norms require ampliative analogical reasoning: no legal agent – human or machine – can know enough subsumption rules to account for the infinite forms the facts of a case can take.[1]

In spite of this, recent approaches in modelling legal reasoning have largely moved away from analogical approaches to rule-based ones[2]. Such approaches have proven to be powerful in capturing the defeasible aspects of the law. However, while it is important to handle defeasibility it is crucial to remember that defeasibility is only necessary as a consequence of inconsistency which is in turn caused by unsound inference.

Humans pay the price of unsoundness to reap a reward: the ability to make ampliative inferences. Contrary to this, present rule-based systems do not offer a corresponding benefit by e.g. providing a mechanism that adds novel rules to their knowledge base. They remain *analytic* in the Kantian sense: any true proposition is already contained in the (skeptical, credulous) closure of their assumptions under the (strict and defeasible) components of their inference system filtered by an appropriate semantic (complete, grounded, ideal, preferred, …).

In a recent *Festschrift* for Erich Schweighofer, Trevor Bench-Capon tongue-in-cheek assures the former (and himself) of his job security as a Good-Old-Fashioned-AI-and-Law ("GOFAIL") researcher by faulting ma-

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[1] We argue these issues at length in our other contribution in this issue: "A Novel Understanding of Legal Syllogism as a Starting Point for Better Legal Symbolic AI Systems.

[2] ATKINSON and BENCH-CAPON, "Reasoning with Legal Cases: Analogy or Rule Application?". In: Proceedings of the seventeenth International Conference on Artificial Intelligence and Law, 2019, pp. 12–21.

chine learning for being inapplicable to case law due to its retrospectiveness (among other things: "When deciding a case, we are not discovering something common to the previous cases, we are *creating* a rule to decide a particular case, and which is intended to constrain *future* cases"[3]. In fairness it should be said that none of the current symbolic GOFAIL methods provide a mechanism to create rules either. In this paper we sketch a path towards such ampliative symbolic AI methods.

We begin by reviewing the literature on analogical reasoning. We then proceed by introducing *context graphs* in the OMDoc/MMT framework. We then provide a sketch of an ampliative inference algorithm operating on context graphs and go through an example in some detail. We conclude by outlining the path ahead towards the implementation of the algorithm sketch.

2. Models of Analogical Reasoning

We can broadly categorize approaches to analogical reasoning[4] along two dimensions: ampliativeness and normativity. Ampliative accounts of analogy count it among the inference modes that allow the generation of "new" knowledge such as induction or abduction. Non-ampliative accounts view analogies as enthymemic (i.e., "gappy") deductive inference. Such enthymemes can be harmless, e.g., the result of omitting trivial minutiae as is often done in technical language among experts; or the enthymeme can be substantial, that is the argument cannot easily be completed, even by an expert. In the latter case non-ampliative analogy rests on the implicit assumption that such a deductive completion is possible. Normative accounts seek to establish reasoning schemes for ideal analogical argument; non-normative (descriptive) accounts aim at discovering the cognitive mechanism behind analogical reasoning.

The most important ampliative and normative theory is the structure mapping theory (SMT) of analogy.[5] It has been implemented in the structure mapping engine (SME) algorithm.[6] SMT judges two domains to be in analogy with each other if an embedding exists between them. The closer to isomorphism (in terms of the theory), the more systematic the embedding, the better the analogy. In the extreme case of an isomorphism the inference becomes deductive. Here the analogy is not seen in the formal structure but rather in shared factual observable properties.[7] Finally Bartha's articulation model[8] focusses on theory construction: the similarities between the source and the target domain must be shown to be critical features of the source and not merely irrelevant ones. Likewise, no critical feature of the target domain may contradict a critical feature of the source domain.

Non-ampliative normative accounts date back to Aristotle's argument scheme from his Rhetoric[9]. In modern times such an approach has been defended by Daves and Russel[10] and Schlimm[11] in his axiomatic account. In these accounts the analogs are instances of an implicit (set of) axiom(s) yielding enthymemic arguments. Notably, the isomorphic special case of structure mapping implies this condition.

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3   Bench-Capon. "The Need for Good Old-fashioned AI and Law". In W. Hotzendorfer, C. Tschohl and F. Kummer (Hrsg.) *International Trends in Legal Informatics: A Festschrift for Erich Schweighofer*. Editions Weblaw: Bern. Pages 23-36, 2020.

4   See Bartha, "Analogy and Analogical Reasoning". In: The Stanford Encyclopedia of Philosophy, 2019 for an overwiev.

5   Forbus, Ferguson, Lovett, and Gentner, "Extending SME to Handle Large-Scale Cognitive Modeling". In: Cognitive Science 41.5, 2017, pp. 1152–1201.

6   Falkenhainer, Forbus, and Gentner, "The structure-mapping engine: Algorithm and examples". In: Artificial Intelligence 41.1, 1989, pp. 1–63.

7   Other theories in this category include the material approaches of Hesse, Models and Analogies in Science, 1966 and Norton, The Material Theory of Induction, 2020.

8   Bartha, By Parallel Reasoning, Oxford University Press, 2010.

9   See Bartha, "Analogy and Analogical Reasoning". In: The Stanford Encyclopedia of Philosophy, 2019 for a discussion.

10 Davies and Russell, "A Logical Approach to Reasoning by Analogy". In: Proceedings of the 10th International Joint Conference on Artificial Intelligence – Volume 1, 1987, pp. 264–270.

11 Schlimm, "Two Ways of Analogy: Extending the Study of Analogies to Mathematical Domains". In: Philosophy of Science 75.2, 2008, pp. 178–200.

However, as Schlimm[12] points out, the inverse does not hold: two domains may obey the same axioms but no isomorphism between them may exist (e.g., the 2-group and the 3-group). Instead, the analogy is established by showing that both analogs obey the same axioms (in the example, the group axioms) or are instances (homomorphic images) of a more complex domain (in the example, the free group). As far as we are aware there are presently no implementations of Schlimm's axiomatic account of analogy.

Finally, descriptive theories[13] attempt to develop cognitively plausible mechanisms for analogical reasoning. They are usually based on connectionist models mimicking human neural processes. Connectionist models may also be highly useful as computational models for the normative theories. Implementations of connectionist models include ACME (Analogical Constraint Mapping Engine)[14], LISA[15] and Copycat[16]. Sowa and Majmudar's[17] VivoMind Analogical Engine claims to straddle SMT and connectionist approaches by implementing SMT "from the bottom up" on conceptual graphs using low-level label and subgraph matchings as well as subgraph transformations to discover high-level structure mappings.

In the legal domain, analogical reasoning has primarily been captured using a version of SMT. The similarity of cases is assessed based on aspects. That is, dimensions of cases that range from maximally pro-defendant to maximally pro-plaintiff and Boolean factors that pertain either to the defendant or to the plaintiff. Conflicting arguments for different possible precedent applications are then constructed based on the degree to which cases resemble each other in terms of their aspects.[18] The main implementations of the analogical approach are the HYPO[19] and CATO systems[20].

In the rule-based approach, precedents are taken to extend the existing body of law by their respective "rule of the case" which is encoded as a defeasible inference rule in the object language of some logic.[21] Defeasibility is established through attacks (undercuts, rebuttals) and preference ordering over rules.

The most important systems include ASPIC+[22] and abstract dialectical frameworks (ADFs) in conjunction with the ANGELIC methodology[23] which allows to encode factor hierarchies as ADFs which can in turn be implemented in a target language such as logic programming.

A (superficially) different approach is taken by the Carneades system.[24] Here links between premises and conclusion instantiate deductive or defeasible argument schemes. For validation, arguments are checked against the catalogue of available schemes.

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[12]   *Ibid.*
[13]   E.g. HOLYOAK and THAGARD. "Analogical mapping by constraint satisfaction". In: Cognitive Science 13.3, 1989, pp. 295–355; MITCHELL, Analogy-making as perception: A computer model. Mit Press, 1993; HUMMEL and HOLYOAK, "A symbolic-connectionist theory of relational inference and generalization". In: Psychological review 110 2, 2003, pp. 220–64.
[14]   HOLYOAK and THAGARD, "Analogical mapping by constraint satisfaction". In: Cognitive Science 13.3, 1989, pp. 295–355.
[15]   HUMMEL and HOLYOAK, "A symbolic-connectionist theory of relational inference and generalization". In: Psychological review 110 2, 2003, pp. 220–64.
[16]   MITCHELL, Analogy-making as perception: A computer model, 1993.
[17]   SOWA and MAJUMDAR, "Analogical reasoning". In: International Conference on Conceptual Structures, 2003, pp. 16–36.
[18]   BENCH-CAPON, "Representing Popov v Hayashi with dimensions and factors". In: Artificial Intelligence and Law 20.1, 2012, pp. 15–35; ATKINSON and BENCH-CAPON, "Reasoning with Legal Cases: Analogy or Rule Application?". In: Proceedings of the Seventeenth International Conference on Artificial Intelligence and Law, 2019, pp. 12–21.
[19]   ASHLEY, Modeling Legal Argument. Reasoning with Cases and Hypotheticals. Artificial intelligence and legal reasoning, 1990.
[20]   ALEVEN, "Teaching Case-Based Argumentation through a Model and Examples", 1997.
[21]   PRAKKEN, "Reconstructing Popov v. Hayashi in a Framework for Argumentation with Structured Arguments and Dungeon Semantics". In: Artificial Intelligence and Law 20.1, 2012, pp. 57–82.
[22]   PRAKKEN and MODGIL. "The ASPIC+ Framework for Structured Argumentation: a Tutorial". In: Argument and Computation 5.1, 2014, pp. 31–62; PRAKKEN, "Reconstructing Popov v. Hayashi in a Framework for Argumentation with Structured Arguments and Dungeon Semantics". In: Artificial Intelligence and Law 20.1, 2012, pp. 57–82.
[23]   AL-ABDULKARIM. "A Methodology for Designing Systems to Reason with Legal Cases using Abstract Dialectical Frameworks". In: Artificial Intelligence and Law 24.1, 2016, pp. 1–49.
[24]   GORDON and WALTON. "The Carneades Argumentation FrameworkUsing Presumptions and Exceptions to Model Critical Questions". In: Proceedings of the 2006 Conference on Computational Models of Argument: Proceedings of Comma, 2006, pp. 195–207.

Rule-based approaches allow to systematically treat the burden of proof and different proof standards through the use of default reasoning and prioritisation of rules respectively. However, Stevens[25] argues that the analogy-account for legal reasoning is preferable as it better accounts for judicial discretion and as a heuristic to find applicable precedents as well as reasons to distinguish a case – especially for novel cases. Aktionson and Bench-Capon[26] lament a lack of computational models for analogical reasoning.

We have already taken a step towards such a model (see Rapp et al. (2020)[27]. There we combined rule-based and analogical reasoning to *represent* analogical reasoning and operationalize the critical questions for analogies established by Stevens. Here we focus on the algorithmic aspects of such reasoning and how it interplays with analytic deductive and argumentative reasoning.

3. Context Graphs in the OMDoc/MMT system

OMDoc[28] is a wide-coverage representation language for mathematical knowledge (formal) and documents (informal/narrative). In the last decade development has focused on the formal aspect leading to the OMDoc/ MMT instance (Meta-Meta-Theories[29] [30]), which increases expressivity, clarifies the representational primitives and formally defines the semantics of this fragment.

OMDoc/MMT is designed to be foundation-independent. I.e., it does not rely on a specific basic language, reasoning system or make any foundation assumptions about the world. Instead, it introduces several concepts to maximize modularity and to abstract from and mediate between different foundations, to reuse concepts, tools, and formalizations. The OMDoc/MMT language *integrates successful representational paradigms*

– the logics-as-theories representation from logical frameworks,

– theories and the reuse along theory morphisms from object-oriented modeling,

– the Curry-Howard correspondence from type theoretical foundations,

– URIs as globally unique logical identifiers from OpenMath,

– the standardized XML-based interchange syntax of OMDoc,

and makes them available in a single, coherent representational system for the first time. The combination of these features is based on a small set of carefully chosen, orthogonal primitives in order to obtain a simple and extensible language design. These primitives are

1. *constants* with optional types and definitions,

2. types and definitions of constants are *objects*, which are syntax trees with binding, using previously defined constants as leaves,

3. *theories*, which are lists of constant declarations and

4. *theory morphisms*, that map declarations in a domain theory to expressions of a target theory.

Using these primitives, logical frameworks, logics and theories within some logic are all uniformly represented as OMDoc/MMT theories, rendering all of those equally accessible, reusable and extendable. Constants, functions, symbols, theorems, axioms, proof rules etc. are all represented as constant declarations, and all terms which are built up from those are represented as objects.

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[25] STEVENS, "Reasoning by Precedent – Between Rules and Analogies". In: Legal Theory 24.3, 2018, pp. 216–254.

[26] ATKINSON and BENCH-CAPON, "Reasoning with Legal Cases: Analogy or Rule Application?". In: Proceedings of the seventeenth International Conference on Artificial Intelligence and Law, 2019, pp. 12–21.

[27] RAPP, ADRIAN and KOHLHASE, Context Graphs for Legal Reasoning and Argumentation, Proceedings of the Third International Workshop on Systems and Algorithms for Formal Argumentation co-located with the 8th International Conference on Computational Models of Argument (COMMA), 2020, p. 56–67.

[28] KOHLHASE, OMDoc – An open markup format for mathematical documents, 2006.

[29] RABE and KOHLHASE, "A Scalable Module System". In: Information & Computation, 2013, pp. 1–54; HOROZAL, KOHLHASE, and RABE, "Extending MKM Formats at the Statement Level". In: Intelligent Computer Mathematics, 2012, pp. 65–80.

[30] RABE, "How to Identify, Translate, and Combine Logics?". In: Journal of Logic and Computation 27.6, 2017, pp. 1753–1798.

Theory morphisms represent truth-preserving maps between theories. Examples include theory inclusions, translations/isomorphisms between (sub)theories and models/instantiations (by mapping axioms to theorems that hold within a model), as well as a particular theory inclusion called meta-theory, that relates a theory to a theory on a higher level on which it depends. This includes the relation between some low-level theory (such as the theory of the legal domain, the world, or the facts of a legal case) to its underlying foundation (such as first-order logic), and the latter's relation to the logical framework used to define it – e.g. LF[31].

All of this naturally gives us the notion of a theory graph, which relates theories (represented as nodes) via vertices representing theory morphisms (as in Figure 1), being right at the design core of the OMDoc/MMT language. It is a central advantage of the OMDoc/MMT system that theory morphisms "transport axioms, definitions, theorems, ... " to new contexts and thus induce knowledge that is not explicitly represented in the graph. Therefore, it is a central design invariant of the system that we can name all induced objects with canonical URIs, the MMT URIs, which contain enough information to reconstruct the induced objects themselves – given the graph.
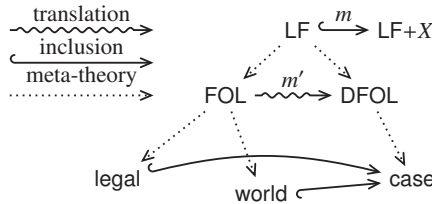

Figure 1: Theory Graph with Meta-Theories

In this setting, meaning trickles down from the urtheories – the only ones that need to be directly implemented in the underlying MMT system – via the logics to the domain level. This gives us a choice of foundations – e.g. between first-order logic (FOL) for descriptions of the world and a defeasible logic (DFOL) for arguing about the facts of a case. MMT comes with a large atlas [32] with hundreds of classical and non-classical logics modularly represented as MMT meta-theories.

One of the salient features of MMT we exploit for context graphs is that theories and morphisms form a category with co-limits, and that MMT can compute pushouts automatically. In a pushout, the situation is as shown on the right: An inclusion $P \hookrightarrow C$ and a view $P \rightsquigarrow S$ together induce the pushout $S \hookrightarrow V \hookleftarrow C$. We call configurations $P \hookrightarrow B$ *functors* as they can be "applied" to an argument theory A over an application view $P \rightsquigarrow A$. We indicate pushout constructions in diagram by placing a pushout symbol into the commuting square pointing to the computed result theory V. Pushouts give us a very structured way of encoding knowledge application in diagrams. As we use the propositions-as-types encoding of assertions, the application views naturally encode the proof obligations that encode the applicability of the functor.
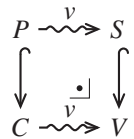
Context graphs are a subcategory of theory graphs in which the theories are internally consistent theories of some base logic. They can be interpreted as belief sets or possible worlds. In context graphs we model legal syllogisms involving subsumption as pushouts: the functor corresponds to the major premise of the syllogism: it consists of a theory of legal conditions and a theory of legal consequences. The minor premise is modelled as the view *v*: it proves that all conditions from *P* are instantiated by the subsumed theory *S*. If this is the case, then the legal consequences – as instantiated via *v* follow and are added to the graph as a new theory *V.*

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[31]   See Pfenning. "Logical Frameworks". In: Handbook of Automated Reasoning, 2001 for an overview.
[32]   CODESCU, HOROZAL, KOHLHASE, MOSSAKOWSKI and RABE, "Project Abstract: Logic Atlas and Integrator (LATIN)". In: Intelligent Computer Mathematics, 2011, pp. 289-291; The LATIN Logic Atlas, url: www.gl.mathhub.info/MMT/LATIN; LATIN2 – Logic Atlas Version 2. url: www.gl.mathhub.info/MMT/LATIN2.

In addition, context graphs are equipped with a defined theory relation called *attack*. Attacks can be defined in terms of diagrams consisting of theories, views and certain features of the base logic. As such they require no additional expansion of the framework[33].

Finally, an important MMT-feature for our purposes is the *theories-as-types* approach. This enables to convert theories into types that in turn can be assigned to constants as they appear within in theories.[34] In this way legal concepts can be nested within each other and expanded as needed.

4. Towards an Analogy-based, Ampliative Legal-Inference System

Consider the following scheme of legal syllogism:

1. Major Premise ("Obersatz"): A is defined in terms of $B_1, \ldots, B_n$;
2. Minor Premise ("Untersatz"): C satisfies $B_1, \ldots, B_n$;
3. Conclusion ("Schlussatz"): C is an instance of A.

This scheme is used in law application/subsumption to arrive at legal decisions. We model it using analogical reasoning. As such it is *not* merely a logical inference rule. Rather, its susbsumtion component is to be modelled as with an inference algorithm that extends the context graph as needed. It consists of three components:

– A logical inference system operating on *legal propositions* formalized in the base logic.
– An analogical inference system operating on *contexts/theories*.
– An argumentative inference system operating on *legal arguments*, i.e., sequences of logical and analogical inferences.

The logical inference system is deductive and analytic. Its derivations are strict proofs. The analogical inference system is ampliative and defeasible: it adds new subsumption rules (and attacks) in the form of views to the context graph and uses them to "push out" the coverage of a given major premise to a new domain. It uses heuristics to find view candidates and then calls the logical inference system to discharge any proof obligations that arise. In this way the view serves a guide to compute those parts of the logical closure that are needed to establish subsumption of the theory to be subsumed. Thus, the combination of the two systems yields defeasible arguments in favor of (or against) subsuming one context under another.

The strength of the argument in favor of the new subsumption determines whether the addition to the knowledge base and hence the syllogism application succeeds. This determination in turn requires the construction of syllogistic arguments for and against the subsumption rule candidate, yielding a recursive process that terminates once it reaches "ground truth", i.e., a syllogism that proceeds purely deductively from premises and existing subsumption rules.

Finally, the argumentative system enforces consistency: whenever the logical and analogical inference systems together allow the derivation of conflicting results it forces a decision for one of the options.

The logical inference system is obtained through automatic or interactive prover support for the respective base logic. Extending MMT with such prover support is ongoing work. The analogical inference system is based on the view finder presented in Müller et al. (2018)[35] . It would need to be extended by powerful heuristic and possibly subsymbolic methods to generate more complicated subsumption candidates. As attack

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[33] For details see: Rapp, Adrian and Kohlhase, Context Graphs for Legal Reasoning and Argumentation, Proceedings of the Third International Workshop on Systems and Algorithms for Formal Argumentation co-located with the 8th International Conference on Computational Models of Argument (COMMA), 2020, p. 56–67.

[34] Müller, Rabe and Kohlhase, Theories as Types. In: Proceedings of the 9th International Joint Conference on Automated Reasoning (IJCAR), 2018, p.575–590.

[35] D. Müller, M. Kohlhase, and F. Rabe, "Automatically Finding Theory Morphisms for Knowledge Management". In: Intelligent Computer Mathematics. Ed. by F. Rabe, W. M. Farmer, G. O. Passmore, and A. Youssef. LNAI 11006. Springer, 2018. doi: 10.1007/978-3- 319-96812-4.

is a relation defined in terms of views, the same view finding techniques can also be applied to find attacks. Finally, the argumentation component is based on the usual abstract argumentation semantics [36].

All of the inference systems may in principle be undecidable: not every logical consequence of the propositions may be inferable by the logical inference system; the analogical inference system – being ampliative – is necessarily undecidable; finally, the argumentative system may not be able to find all attack relations between arguments.

This being said, the systems are sound in the following sense: every logical derivation found or certified by the system is correct; every subsumption found or certified by the system is syntactically correct; thus, every argument found or certified by the system is a correct legal argument. Finally, every attack between arguments found or certified by the system is a correct attack.

5. Example: The Allowance Case (Taschengeld-Fall)

We will illustrate our approach by using the following legal practice problem as an example. Even though it is very simple – it is one of the first problems budding legal students are given to sharpen their legal reasoning skills – it already shows many legal complexities.

Taschengeldfall: Der 16-jährige Fabian hat wegen einer längeren Krankheit sein wöchentliches Taschengeld nich ausgeben können und daher 50€ angespart. Da er nichts von „Anlegen für schlechte Tage" hält, will der das Geld sofort ausgeben. Im Spielwarengeschäft des M entdeckt er eine täuschend echt aussehende Spielzeugpistole, mit der er seinen Freunden mal so richtig „einheizen" will. Auf Nachfrage des M teilt F diesem mit, dass er die Pistole von seinem Taschengeld kaufe und dass seine Eltern hiermit auch einverstanden seien. Dabei hatten F's Eltern diesem explizit verboten „solchen Unfug" zu kaufen. Waffen, wenn auch nur Spielzeugwaffen, hätten in ihrem Haus nichts zu suchen. Die gekaufte Pistole versteckte F in seinem Zimmer, wo sie seine Mutte sehr zu ihrem Missfallen am nächsten Tag fand. F's Eltern verlange von M die Rückerstattung des Kaufpreises gegen Rückgabe der Pistole. Dies lehnt M ab. Immerhin hätte die Pistole ja deutliche Gebrauchsspuren.

Bearbeitungsvermerk: Er ist davon auszugehen, dass M das Geld noch hat und gesondert aufbewahrt. Frage: Kann F – vertreten durch seine Eltern – von M Rückzahlung des Kaufpreises gegen Rückgabe der Pistole verlangen?

This little "case" has two possible solutions: one argues via §985 BGB (claim to return possession of the bills/money, which is still owned by the minor F) and another via §812 Alt. 1 BGB (claim to pay back the purchase price); both conclude that M needs to recompense F. A schematic visualization of the structure of the first argument is given on the left of Figure 2. On the right we show an OMDoc/MMT theory graph that implements the same structural intuitions in an MMT context applying the method put forward Rapp et al (2020)[37] to the allowance case. Figure 2b thus could be the result of an application of the intended inference system to the allowance case.

Procedurally, we expect that an inference algorithm would follow the nesting in Figure 2a. Hence it starts with the "facts"-theory in the right upper corner of Figure 2b. The analogical system is called to find subsumption candidates in the existing context graph formalization of the law. E.g., in this case it might find §985 (the two boxes below the facts theory; technically, we have to first apply a temporal default rule). §985 requires an owner which is instantiated by the boy Fabian in this example. However, that subsumption itself has to be

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[36] Dung, On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming, and n–person games. Artificial Intelligence. 77 (2): 321–357.

[37] Rapp, Adrian and Kohlhase, Context Graphs for Legal Reasoning and Argumentation, Proceedings of the Third International Workshop on Systems and Algorithms for Formal Argumentation co-located with the 8th International Conference on Computational Models of Argument (COMMA), 2020, p. 56–67.
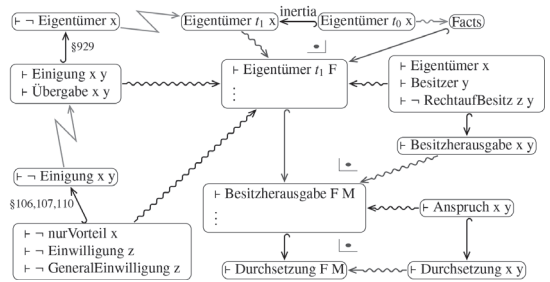
justified which is again done through the construction of syllogistic arguments for and against. Arguments for proceed by discharging the proof obligations of the subsumption candidate through the logical system and elaborating the predicate (or type) "*Eigentümer*" to its corresponding theory, in turn mapping its declarations to the facts and so forth.

Thus the reasoning process moves in the manner of definition unpacking until either further proof or view finding fails or all proof obligations can be discharged. Some boxes can be evaluated "lazily", e.g. the statute of limitations (Anspruch erloschen?) may have a default value and may only need to be checked in reaction to a challenge (burden of proof).

An argument against the application of §985 can be constructed from §929 which states that the original owner of the money loses his property in it if he transfered it in a valid transfer agreement. Hence if F entered in a valid transfer agreement, he cannot be the owner of the money anymore. At this level, further conflicting subsumption candidates arise: on one side it can be argued that the declaration of intent (*Willenserklärung*) of the minor is valid because his intention (§110) or the legitimitate expectation of the seller should be legally relevant or it is invalid because the objection of the minor's parents (to his purchase of the toy gun) should take legal priority (§§106, 107 and Article 6 GG). In fact, the parents win in this case due to Article 6 GG: they can mandate how the son can use his allowance. Hence the outcome of this argumentation process (which is collapsed in Figure 2b) is an attack on the application of §929.
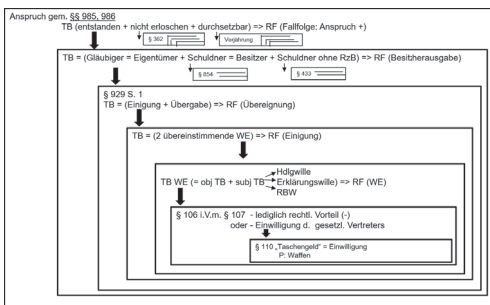
(a) The Matroschka Diagram   (b) The MMT Theory Graph

Figure 2: Analysis of a case in terms of nested subsumtions of norms

Thus the initial rule application stands and F indeed remained the owner of the money. After this there are two further functor applications to the thusly modified facts, eventually yielding the result that F has an actionable claim to M. The further details pertaining to checking whether F claim may have expired are ommitted from this diagram for simplicity.

6. Conclusion

In prior work we have presented a system to represent and check analogical and argumentative reasoning. Here we presented a road map towards extending this towards inference. We have presented an outline of an ampliative, analogical inference algorithm on context-graph-based representations of legal knowledge. Implementing this algorithm will require work on two fronts: firstly, careful fine tuning of the representation formalism to ensure powerful yet decidable logical and type inference wherever possible. Secondly, building on our existing work on view finding to enable the analogical inference component of the system. Currently, views can only be found if they don't require theorem proving. In the future we want to use prover integration to find more complex views and thereby subsumption candidates and attacks. Indeed, it may even be possible to leverage the prover as a reward component in a reinforcement learning system that is trained to find view

candidates. The prover would attempt to verify the candidates generated by the learner and provide positive feedback if this succeeds.

7. References

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